Chapter 1 - Instabilities and Moist Convection

Michael J. Reeder

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1.1 Introduction

Predicting rainfall in the tropics remains one of the most *important* and *challenging* problems in meteorology. The problem is *important* for a number of reasons. First, water is central to life on our planet, and too much or too little water can be catastrophic for people, society and the environment. Second, the important weather systems in the tropics are, in general, existentially inter-dependent with deep precipitating clouds. And third, these deep precipitating clouds heat the tropical atmosphere, driving wind systems that span much of the planet. The problem is *challenging* because models, ranging from those used in numerical weather prediction to those used for climate, generally handle tropical rainfall relatively poorly and it is not fully understood why.

Introductory meteorology textbooks teach us that when air rises without external heating or cooling, it expands and its temperature falls until the air is saturated, at which point water vapour begins to condense. The associated heating from condensation reduces the rate at which the temperature decreases, so that if the air continues to rise, its temperature falls less rapidly than it would otherwise. Consequently, the rising air may become warmer and hence more buoyant than its immediate surroundings, in which case the air continues to rise and a cloud is born (Figure 1.1a). This kind of instability is called *conditional instability* and is the central topic of this chapter. If the cloud continues to grow, precipitation may form (Figure 1.1b).

As liquid water evaporates or ice melts the atmosphere is cooled and moistened, which is another important physical process associated with precipitating clouds (Figure 1.1b). As the cloud matures, some of the precipitation evaporates in the unsaturated air below the cloud producing cold downdraughts at low levels. These downdraughts spread out along the surface as a cold pool (or gravity current), with a gust front marking the leading edge. At times, as this pool of cold air spreads, it blocks the source of warm moist air and the cloud dies (Figure 1.1c). At other times, it lifts the low-level warm moist air, maintaining the cloud or giving birth to new clouds. This possibility is explored in later chapters.

In meteorology, the vertical overturning circulations driven by *buoyancy* are called *convection*. Importantly, convection transports mass and other properties, such as water, vertically. Although the definition of buoyancy is not unique, broadly speaking, it is a vertically-oriented force (usually expressed per unit mass) resulting from the difference between the weight of a volume of air and the weight of a neighbouring volume. The buoyancy driving the convection can be produced by physical processes such as surface sensible heating, in which case the overturning circulation is called *dry convection*. The main example of dry convection in the atmosphere is in the boundary layer. Here, vertical overturning circulations transport heated air from the near surface aloft and transport relatively cooler air from aloft to the near surface. In contrast, the buoyancy driving the overturning circulation could be a result of the change of phase of water, as in the example of conditional instability describe above. These two types of convection, dry and moist, are very different. For example, in the case of dry convection, the ascending

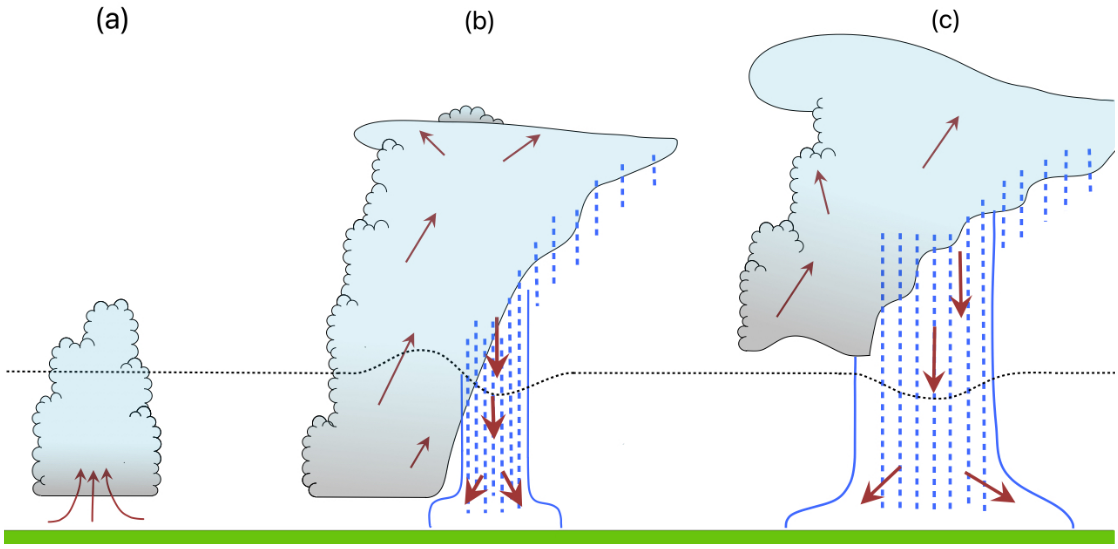

Figure 1.1: Thunderstorms and other organized convective systems are generally a collection of individual convective cells that grow and decay on timescales of 30 min - 1 hr. The lifecycle of a convective cell is depicted here. (a) Cumulus stage: a convective cloud is born through lifting and saturation. (b) Mature stage: the cumulus cloud has developed into a cumulonimbus. The cloud rains, and evaporative cooling produces downdraughts and cold pools. (c) Dissipating stage: the spreading cold pool blocks the supply of warm moist air and cloud dies, or perhaps aids in lifting warm moist air, possibly far from the storm, thereby initiating more convective cells. Precipitation is represented by the short vertical dashes. The updraughts and downdraughts are represented by arrows. The lateral extent of the cold downdraughts and the associated surface cold pools are marked by the longer dashed lines in (b) and (c). The freezing level (0^oC isotherm) is marked by a near horizontal dotted line. Adapted from Doswell (1985).

and subsiding branches of the overturning circulation have similar widths and similar speeds, whereas in the case of moist convection, the ascending branch is narrower and faster than and subsiding branch. Importantly, moist convection is marked by cloud and often precipitation.

As the aim of the present chapter is to explore the most basic physics of dry and moist atmospheric convection, only the simplest (and least accurate) versions of the physical arguments and supporting mathematical theory are presented here. In the same spirit, we focus exclusively on the liquid phase of water and ignore the complications of ice. More complete (and more accurate) versions can be found, for instance, in Emanuel (1994).

1.2 Aerological Diagrams

One of the most common ways to check for conditional instability is by using an *aerological diagram*, also known as a *thermodynamic diagram*. Aerological diagrams graphically depict the thermodynamic state of the atmosphere and the various thermodynamic processes by which this state might change. These thermodynamic processes can be easily analysed without long calculations, or the calculations can be clearly presented on such diagrams. The state of the atmosphere is generally taken from radiosonde ascents, although model output is sometimes used. Thermodynamic diagrams are one of the most enduring and basic tools used in *forecasting* convection.

There are several versions of aerological diagrams used. The one used in this chapter is known as a *skew T - log P* diagram (Fig. 1.2). On such a diagram, the ordinate is $\ln p$, where p is the pressure, and the abscissa is the temperature T. (Recall that $\ln p$ is proportional to height in an isothermal atmosphere.) One of the distinctive properties of a skew T - log P diagram is that the isotherms lie at 45^o to the isobars, making the abscissa skew. In other words, the coordinate axes are not orthogonal. The reason for this choice is simply that the resulting graph of the atmospheric temperature profile is more compact.

Observations of T, dew-point temperature T_d and p are plotted on the diagram. (Recall that T_d is the temperature to which air must be cooled at constant pressure for it to become saturated. It is defined later in Section 1.5) The two points (p, T) and (p, T_d) uniquely characterize the state of moist but unsaturated air. (*Moist air* refers to the mixture of dry air and water vapour, neglecting hydrometeors.) Once the air is saturated, only one point (p, θ_e) is required, where θ_e is the *equivalent potential temperature* (which is also defined later in Section 1.6.3).

In what follows, the lines on the aerological diagram and the associated thermodynamic processes will be explained. The two most important thermodynamic processes for convection are (i) saturation by ascent and (ii) cooling and moistening by the evaporation of water. An example of (i) is conditional instability and the formation of convective cells (Fig. 1.1a), while an example of (ii) is the formation of downdraughts and cold

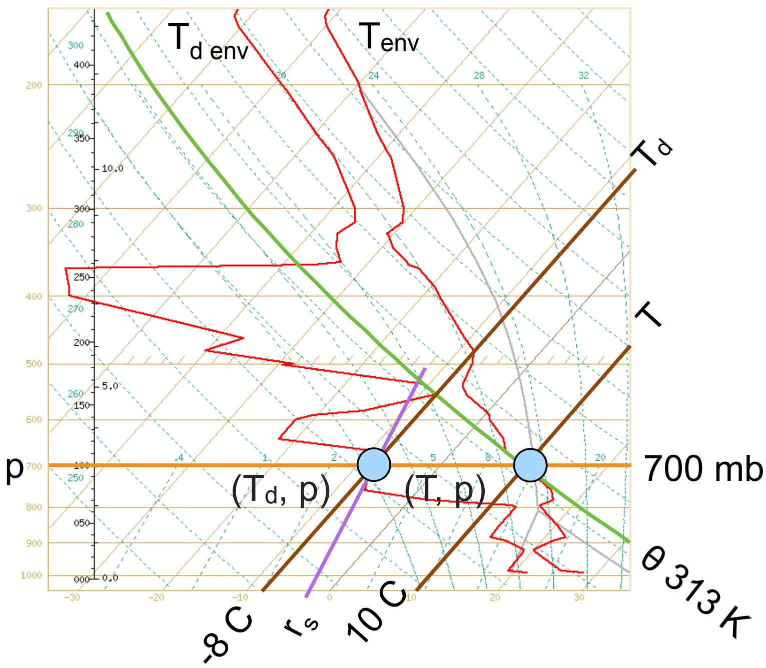

Figure 1.2: Skew T - log P aerological diagram. The ordinate is $\ln p$. Isobars (brown), lines of constant pressure p, are horizontal. Isotherms (thin brown), lines of constant temperature T, are oriented at 45^o to the isobars. Isopleths of saturated mixing ratio r_s (dashed green) are oriented from the bottom left to the top right. Isentropes (dashed green), lines of constant potential temperature θ, also called adiabats, are curved and oriented from the bottom right to the top left. θ is defined by Eq. 1.3. Moist isentropes (dashed green), also called moist-adiabats, are lines of constant (pseudo) equivalent potential temperature θ_e and are curved in the opposite sense to the isentropes, and also oriented from the bottom right to the top left. θ_e is defined in Section 1.6.3. The red line to the right, labelled T_{env}, graphs the environmental observations of temperature. The red line to the left, labelled $T_{d\ env}$, graphs the environmental observations of dew-point temperature. For illustration, the thermodynamic properties of the environment at 700 mb are highlighted. Note: 1 mb = 1 hPa = 100 Pa. The thick orange line marks 700 mb. At this pressure, $T = 10^o$ C, marked by the rightmost thick brown line, and T_d = -8^o C, marked by the leftmost thick brown line. The thick green line is the adiabat corresponding to $\theta = (10 + 273)\,(1000/700)^{\frac{287}{1004}} = 313$ K. The violet line is the isopleth of $r_s = 3$ g kg^{-1} passing through the point (T_d, p) = (-8^o C, 700 mb) and is therefore equal to r.

pools (Fig. 1.1b).

1.3 Ideal Gas Law and First Law of Thermodynamics

This section begins to make some of the ideas of the previous sections more quantitative with the aid of two important physical laws: the *Ideal Gas Law* and the *First Law of Thermodynamics*. In this section, the mixture of gases that constitute air is taken to be free of water vapour; in other words, the air is dry.

1.3.1 Ideal gas law

The Ideal Gas Law (also called the *Perfect Gas Law* or the *Equation of State*) is $p = \rho\, R_d\, T$, and links the pressure, density and temperature, where $R_d = 287$ J K^{-1} kg^{-1} is the *gas constant for dry air*. This law encapsulates such observations as: (i) when ρ is held constant, as in an enclosed can (the mass is constant and the volume is constant and hence ρ is constant), p increases when T increases; and (ii) if p is held constant, such as for short times at the surface of the Earth, cold air is heavy and warm air is light.

1.3.2 First law of thermodynamics

The First Law of Thermodynamics is a statement of conservation of energy. Apart from the macroscopic (averages over sufficiently large patches) kinetic and potential energy, the atmosphere has internal energy by virtue of the motion and position of the constituent atoms and molecules. The kinetic energy of these atoms and molecules is related to the temperature through the *Kinetic Theory of Gas*. For an ideal gas, the internal energy depends only on T, which is known as *Joule's Law*. The rate of change of internal energy with T is the *heat capacity at constant volume* c_v. There are two ways to change the internal energy. The first is by heating or cooling the atmosphere. Examples of this include radiation and changing the phase of water (the evaporation of water cools the atmosphere whereas condensation warms it). The second is by compressing or expanding the gas, which can be expressed in terms of work. Hence, in words, the First Law of Thermodynamics is expressed as: *rate of change of internal energy = rate of heating + rate of work done on the gas*.

Mathematically, the First Law of Thermodynamics is expressed as

$$c_v \frac{dT}{dt} = \dot{q} - p\frac{d\alpha}{dt} \tag{1.1}$$

where $\dot{q}$ is the heating rate per unit mass, and $\alpha = 1/\rho$ is the *specific volume*.

1.3.3 Adiabatic processes and potential temperature

One of the most important thermodynamic process is an *adiabatic process*. This is one for which $\dot{q} = 0$, meaning that the atmosphere is neither heated nor cooled (e.g. through radiation or a change in the phase of water). As an example, consider what happens in an adiabatic process: $d\alpha/dt > 0$ (expansion) implies $dT/dt < 0$ (cooling), and vice versa. Hence, when air rises without external heating or cooling, it expands due a decrease in the pressure and its temperature falls, which is the first step in creating a convective cloud.

Using the Ideal Gas Law, the First Law of Thermodynamics (Eq. 1.1) can be re-expressed as

$$c_p \frac{d}{dt} \ln \theta = \dot{q} \tag{1.2}$$

where

$$\theta = T \left(\frac{p_{ref}}{p} \right)^{\frac{R_d}{c_p}} . \tag{1.3}$$

Here θ is the *potential temperature*, which, according to Eq. 1.2, is conserved (meaning it remains constant) for an adiabatic process. Physically, the potential temperature can be interpreted as the temperature an air parcel would have were the pressure changed adiabatically to p_{ref}. Note that when calculating θ from Eq. 1.3, T is the *absolute* temperature, which has units of Kelvin.

Dry adiabats are curves of constant θ on an aerological diagram. For example, an air parcel with $T = 10^o$C $= 10{+}273$ K and $p = 700$ mb with $p_{ref} = 1000$ mb will follow the thick the green curve on Fig. 1.2 if the process is adiabatic. Curves of constant potential temperature are also called *isentropes*.

Γ is the *lapse rate*, which is defined as $-dT/dz$, and represents the rate at which the temperature *decreases* with height. Differentiating Eq. 1.3 with respect to height gives

$$\Gamma = \Gamma_d - \frac{T}{\theta} \frac{d\theta}{dz}. \tag{1.4}$$

$\Gamma = \Gamma_d = -g/c_p \approx 10^o$ C/km, the *dry adibatic lapse rate*, when θ is constant.

1.4 Newton's Second Law and Buoyancy Force

The most important quantity in any discussion of convection is buoyancy, which is the focus of the present section.

1.4.1 A parcel of air

Before deriving an expression for the buoyancy, we introduce the idea of an *air parcel*. A parcel is a volume enclosing a sufficiently large number of atoms and molecules that the density ρ, pressure p and temperature T of the volume are statistically well defined, yet small enough that these statistical properties can be taken to be a constant in the volume. The idea of a parcel is important as, when checking for instabilities, we conduct a thought experiment wherein a parcel is displaced. Whether or not the resultant force on the parcel is in the same or opposite direction as the displacement determines whether or not the atmosphere is unstable.

1.4.2 Forces on a parcel of air

We turn our attention now to the buoyancy and begin by investigating the forces acting in the vertical on a parcel of air. To make the calculations simpler, assume the parcel is box-shaped with volume $A\,\Delta z$, where A is the horizontal area of the box and Δz is the vertical distance (Fig. 1.3), although the following conclusions hold true for a parcel of any shape. Suppose that the density of the parcel ρ_{par} is different (for some reason) to the density of the air in the immediate environment at the same height ρ_{env}. Then, in the vertical, there are three forces acting on the box: a pressure force on the top face $-p_{env}(z+\Delta z)\,A$, where $p_{env}(z+\Delta z)$ is the environmental pressure at height $z+\Delta z$, a pressure force on the bottom face $p_{env}(z)\,A$, and the weight $-\rho_{par}\,g\,A\,\Delta z$. (Recall that the weight of a body is the force on that body due to gravity, and hence, weight = mass $\times$ g.) Finally, p_{env} and ρ_{env} are related to each other through hydrostatic balance $dp_{env}/dz = -g\rho_{env}$.

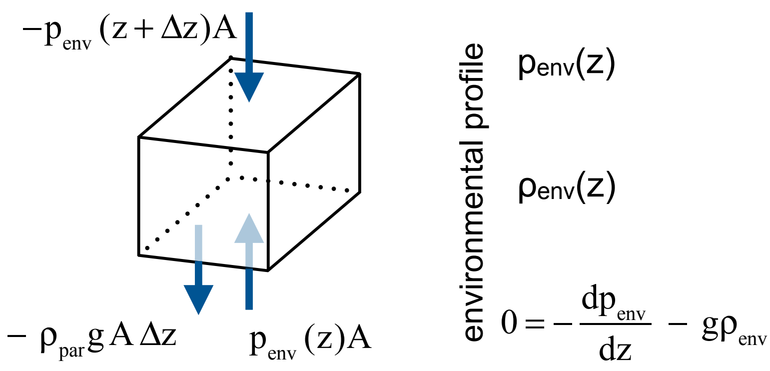

Figure 1.3: Forces acting on a box-shaped parcel of air with mass $\rho_{par}\,A\,\Delta z$. The forces are: a pressure force on the top face $-p_{env}(z+\Delta z)\,A$, a pressure force on the bottom face $p_{env}(z)\,A$, and the weight $\rho_{par}\,g\,A\,\Delta z$. Here p_{env} is the environmental pressure and ρ_{env} is the environmental density, which are related to each other through hydrostatic balance $dp_{env}/dz = -g\rho_{env}$. Upward-acting forces are positive and downward-acting forces negative.

Newton's Second Law states that: *mass* $\times$ *acceleration* $=$ *net force*. When applied to the parcel, Newton's Second Law can be expressed mathematically as

$$\rho_{par}\, A\, \Delta z\, \frac{d}{dt} w_{par} = -p_{env}(z+\Delta z)\, A - \rho_{par}\, g\, A\, \Delta z + p_{env}(z)\, A \tag{1.5}$$

where w_{par} is the vertical component of the velocity, and dw_{par}/dt is the component of the acceleration in the vertical. After using hydrostatic balance in the environment, expanding $p_{env}(z+\Delta z)$ in a Taylor series, and taking the limit as the box becomes infinitesimally small, Eq. 1.5 becomes

$$\frac{d}{dt} w_{par} = b = -g \left(\frac{\rho_{par} - \rho_{env}}{\rho_{par}} \right) \tag{1.6}$$

where b is the *parcel buoyancy*.

The right hand side of Eq. 1.6 can be interpreted physically as the force due to the difference between the weight of the environmental air that would have been there had it not been displaced by the parcel and the weight of the parcel, divided by the mass of the parcel. This result is also known as *Archimedes Principle*. When the density of the parcel is larger than the surrounding density ($\rho_{par} > \rho_{env}$), the buoyancy force (per unit mass) is negative ($b < 0$). Conversely, when $\rho_{par} < \rho_{env}$, $b > 0$ and hence there is an upward force on the air parcel.

In meteorology, it is usually more convenient to express the state of the atmosphere in terms of potential temperature than density. Assume that the pressure of the parcel equals the pressure of the environment: were this not true, the implied pressure gradient force would presumably drive a circulation that would bring the pressures into equilibrium. Hence, $p_{par} = p_{env} = \rho_{par}\, R_d\, T_{par} = \rho_{env}\, R_d\, T_{env}$, which implies that $\rho_{env}/\rho_{par} = T_{par}/T_{env}$. Hence Eq. 1.6 becomes

$$b = g \left(\frac{T_{par} - T_{env}}{T_{env}} \right) = g \left(\frac{\theta_{par} - \theta_{env}}{\theta_{env}} \right) \tag{1.7}$$

where the step to the final expression on the right hand side has used Eq. 1.3. An additional complication of importance to atmospheric convection, but neglected here, is that the hydrometeors (such as rain water droplets and ice) contribute to the buoyancy. For example, see Emanuel (1994).

1.4.3 Static stability

We are in a position now to assess the stability of a dry atmosphere to vertical displacements. Consider the buoyancy of a parcel of air in the environment lifted adiabatically from height z to height $z + \Delta z$. As the potential temperature is conserved,

$\theta_{par}(z + \Delta z) = \theta_{par}(z) = \theta_{env}(z)$. Then, from Eq. 1.7, the parcel buoyancy at $z + \Delta z$ is

$$b = g \left(\frac{\theta_{env}(z) - \theta_{env}(z + \Delta z)}{\theta_{env}(z + \Delta z)} \right) \approx - \left(\frac{g}{\theta_{env}} \frac{d\theta_{env}}{dz} \right) \Delta z = -N^2 \, \Delta z \tag{1.8}$$

where N is the *Brunt-Vaisala frequency*, also called the *buoyancy frequency*.

The important point here is that the parcel buoyancy, and hence the stability of the environment to vertical displacements, can be determined solely from a knowledge of the environmental potential temperature. When $N^2 > 0$, b and Δz have opposite signs, and consequently the buoyancy force acts to accelerate a displaced parcel towards its original height. In this case, the atmosphere is said to be *statically stable*. Conversely, when $N^2 < 0$, b and Δz have the same sign, and consequently the buoyancy force acts to accelerate the parcel away from its initial height. In this case the atmosphere is *statically unstable*.

The condition for the static stability of the atmosphere is commonly expressed in terms of the *environmental* lapse rate

$$\Gamma_{env} = \Gamma_d - \frac{T_{env}}{\theta_{env}} \frac{d\theta_{env}}{dz}. \tag{1.9}$$

which is Eq. 1.4 applied to the environment. Neutral static stability, $d\theta_{env}/dz = 0$, implies that $\Gamma_{env} = \Gamma_d$ and hence the atmosphere cools at the adiabatic lapse rate. Static *stability*, $d\theta_{env}/dz > 0$, implies that $\Gamma_{env} < \Gamma_d$, as both θ_{env} and T_{env} are positive, and hence the atmosphere cools less quickly than the adiabatic lapse rate. Likewise, static *instability*, $d\theta_{env}/dz < 0$, implies that $\Gamma_{env} > \Gamma_d$ and hence the atmosphere cools more quickly than the adiabatic lapse rate. In this case, the lapse rate is said to be *super-adiabatic*.

Mixing in the convective boundary layer (where the buoyant production of turbulence is much larger than the shear production) is accomplished by thermals, which are discrete volumes of buoyant fluid. Thermals are produced when the surface is heated through daytime solar radiation and becomes sufficiently hotter than the overlying air. The result of mixing, or *dry convection*, is to produce a layer in which the potential temperature is constant and the buoyancy zero. Such a layer is known as a *well-mixed layer*.

1.5 Measures of Water Content and Saturation

The previous section dealt with dry thermodynamic processes. As our aim now is to examine moist convection, a few relevant measures of the water content in the atmosphere and saturation are discussed briefly.

Dalton's Law states that the pressure of a mixture of gases is the sum of the pressures exerted by each constituent gas individually; these individual pressures are called *partial pressures*. Mathematically, Dalton's Law is $p_{total} = p + e$, where p_{total} is the total pressure, p is the partial pressure of dry air, and e is the partial pressure of water vapour, called the *vapour pressure*. (In the preceding sections, p_{total} and p represent the same quantity as $e = 0$.) Like dry air, water vapour satisfies its own version of the Ideal Gas Law, which is $e = \rho_v \, R_v \, T$, where ρ_v is the density of water vapour, and $R_v = 461$ J K^{-1} kg^{-1} is the *gas constant for water vapour*.

The most common measure of water vapour content is the *water vapour mixing ratio* r, which is the ratio of the mass of water vapour m_v to the mass of dry air m

$$r = \frac{m_v}{m} = \frac{\rho_v}{\rho} \approx \frac{\epsilon e}{p}$$

where $\epsilon = R_d/R_v \approx 0.622$.

From the Ideal Gas Laws for dry air and water vapour

$$p = \rho \, R_d \left(\frac{T}{1 - r\left(\frac{1-\epsilon}{\epsilon}\right)} \right).$$

The right bracketed term is the *virtual temperature* $T_v \approx T(1 + 0.61r)$. When written in terms of T_v, the above equation for a mixture of dry air and water vapour looks the same as that for dry air, and uses a single gas constant R_d. Physically, T_v is the temperature required for an equivalent volume of dry air to have the same density as the moist air parcel. Since $r \geq 0$ it follows that $T_v = T(1 + 0.61r) \geq T$. In the previous section (Section 1.4), the discussion of the buoyancy assumed dry air. However, all the results carry over to moist air provided (i) the air is unsaturated, and (ii) T is replaced with T_v, including in the definition of the potential temperature (which is then the *virtual potential temperature* θ_v).

Although up to now the terms *saturated* and *unsaturated* have been used freely, it is time to define their meanings more precisely. An enclosed volume of air lying over a flat water surface is said to be *saturated* when condensation on to the surface and evaporation from the surface are in equilibrium. At saturation, $e = e_s$, which is called the *saturation vapour pressure*. The expression describing how this equilibrium vapour pressure varies with temperature is not derived here, but can be found in standard textbooks such as Emanuel (1994). The expression is known as the *Clausius-Clapeyron Equation* and is written

$$\frac{de_s}{dT} = \frac{Le_s}{\alpha_v - \alpha_l} \approx \frac{Le_s}{R_v T^2} \tag{1.10}$$

where α_l is the specific volume of liquid water, α_v is the specific volume of water vapour, and $L = 2.5 \times 10^6$ J kg^{-1} is the *latent heat of vaporisation*. The step from the second to third terms in Eq. 1.10 makes use of the inequality $\alpha_l \ll \alpha_v = (R_v T)/e_s$. Equation 1.10 can be integrated with respect to T to give

$$e_s(T) = e_{s0} \exp\left(-\frac{L}{R_v}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right) \tag{1.11}$$

where $e_{s0} = 6.112$ mb and $T_0 = 273.16$ K are the values of saturation pressure and temperature at the *triple point*, which thermodynamic sate in which the three phases of water coexist. A commonly used accurate empirical fit to Eq. 1.11 from Bolton (1980) is

$$e_s(T) = 6.112 \exp \frac{17.67\,T}{T + 243.5}$$

where, in this expression, the unit for e_s is mb and the unit for T is ^{o}C.

The *dew point* T_d is the temperature at which the saturation vapour pressure equals the actual vapour pressure, $e_s(T_d) = e$. The *saturation mixing ratio* r_s is $r_s \approx \epsilon\, e_s/\,p$, lines of which are plotted on an aerological diagram (Fig. 1.2).

1.6 Conditional Instability

Conditional instability is the atmospheric instability responsible for moist convection and it is the focus of the present section.

1.6.1 Saturation and instability by ascent

As mentioned earlier, an unsaturated air parcel requires two points on an aerological diagram to define its thermodynamic state. For a parcel at 1000 mb in Fig. 1.4, those two points are A and B. Point A is defined by p_{1000} and T_{1000}, whereas point B is defined by p_{1000} and $T_{d\,1000}$. Point B provides information on how close the air is to saturation. Note that by definition $r(p_{1000}, T_{1000}) = r_s(p_{1000}, T_{d\,1000})$.

Suppose now the parcel is lifted by the atmospheric circulation. As it ascends, θ is conserved, and hence point A follows a dry adiabat (thick green line). As the mass of water vapour in an unsaturated parcel is conserved, r is also conserved, and the point B follows a line of constant r_s starting from $T_{d\,1000}$.

The parcel cools at the dry adiabatic lapse rate as it ascends, and at some point condensation begins. As the parcel is now saturated, only one point is needed to define its thermodynamic state. This is point C in Fig. 1.4 and it is called the *Lifting Condensation Level* (LCL). If the parcel continues to be lifted by the atmospheric circulation, the parcel is heated to some degree as the water condenses, and hence the rate at which the parcel cools is less than the dry adiabatic lapse rate. (It cools at the *moist adiabatic lapse rate*.) Moreover, the parcel follows a pseudo-adiabat, which is defined below in Section 1.6.3. An exact expression for the LCL can be found in Romps (2017).

The parcel is colder than its environment, and hence negatively buoyant. Work must be done by the atmosphere to lift the parcel until it reaches the *Level of Free Convection* (LFC). This is point D in Fig. 1.4, after which the parcel is warmer than its environment, and hence positively buoyant. The parcel no longer requires the atmospheric circulation to do work to lift it.

The parcel remains warmer than its environment until it reaches point E in Fig. 1.4. This is the *Level of Neutral Buoyancy* (LNB), also called the *Equilibrium Level*. The parcel generally overshoots the LNB, as it has inertia, and becomes negatively buoyant.

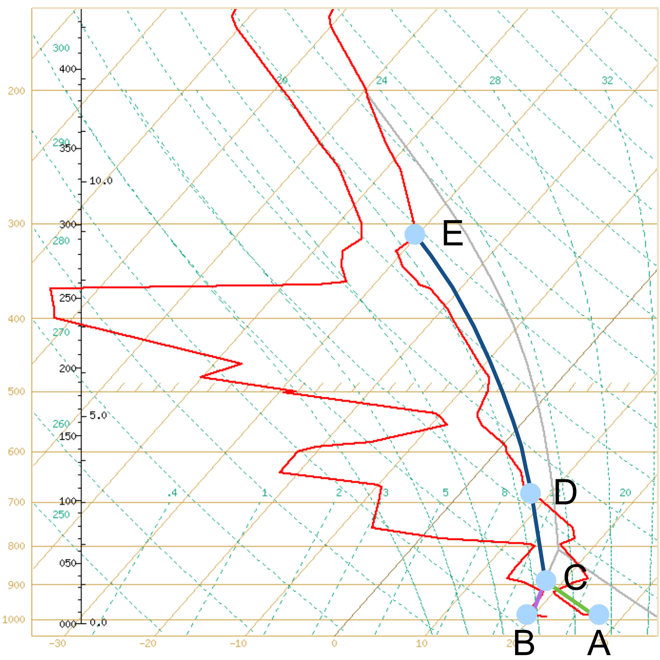

Figure 1.4: Conditional instability. The Skew T - log P aerological diagram and temperature sounding from Fig. 1.2. The thermodynamic state of a near surface air parcel is defined by A(p_{1000}, T_{1000}) and B($p_{1000}, T_{d\,1000}$). It is lifted to C(p_{LCL}, T_{LCL}), the Lifting Condensation Level, at which point condensation begins. It is lifted further to D(p_{LFC}, T_{LFC}), the Level of Free Convection (LFC), after which the parcel is positively buoyant and freely ascends. Beyond E(p_{LNB}, T_{LNB}), the Level of Neutral Buoyancy (LNB), the parcel becomes negatively buoyant and consequently decelerates.

1.6.2 Mixed layers and the initiation of moist convection

That work must be done on the parcel to lift it before it becomes positively buoyant was emphasised in the foregoing discussion of conditional instability. However, recall from Eq. 1.8 that when the potential temperature is constant with height, no buoyancy is produced when parcels are lifted, and consequently, no work is required to lift a parcel. (This is, of course, an idealisation, but it is nonetheless substantially true.)

The surface is heated during the day by solar radiation, and the heated air in contact with the surface is mixed through the overlying atmosphere by thermals, producing a layer in which the potential temperature is constant. This layer is called a *mixed layer*. If the mixed layer is deep enough, cumulus clouds form, and the height at which they form is called the *Convective Condensation Level* (CCL). The CCL is calculated on an aerological diagram as the intersection point of r_s and the environmental temperature profile (Fig. 1.5). The surface temperature at which the CCL is reached is called the *Convection Temperature* (CT) and is determined by adiabatically bringing a parcel from the CCL to the surface.

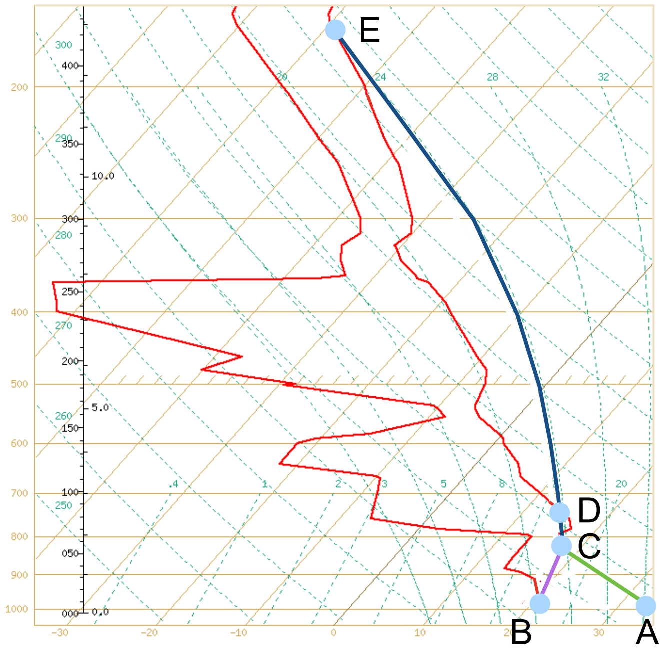

Figure 1.5: Convective Condensation Level (CCL). The CCL is the intersection of the environmental temperature profile (rightmost red line) and the line B-C, which is the isopleth of r_s starting from $T_{d\,1000}$ (thick violet line). The temperature at intersection of the adiabat C-A with the surface (taken here to be 1000 mb) is the Convection Temperature (CT).

1.6.3 Pseudo-adiabatic processes and equivalent potential temperature

The *equivalent potential temperature*, is a modification of the potential temperature that takes account of the effect of condensation. It is useful in analysing the thermodynamics of an ascending or descending air parcel as it is conserved in both saturated and unsaturated conditions. The isopleths of constant θ_e go by a variety of names, including *moist isentropes*, *moist adiabats* and *pseudo-adiabats*. Here we will use the term pseudo-adiabat. As an example, the thick blue line in Fig. 1.2 is the pseudo-adiabat for a saturated parcel at $p = 700$ mb with $T = 10^o$ C. A second example is Fig. 1.4; here, the ascending parcel becomes saturated at the LCL and subsequently follows the thick blue curve, which is a pseudo-adiabat. As a third example, the thick blue line in Fig. 1.5 is the pseudo-adiabat followed by a parcel beyond its CCL.

Physically, θ_e for a given p and r_s, is the value of θ after all the water vapour is condensed and used to heat the air (Fig. 1.6). Such a thermodynamic process, in which the condensed water is removed, is called *pseudo-adiabatic* and the value of θ_e calculated this way is called the *pseudo-equivalent potential temperature*, although here we drop the prefix for simplicity. The process is *pseudo-adiabatic* because the rain removes some heat from the cloud. (If all the liquid water remains in the rising parcel as cloud droplets with no rain, the process is reversible and called *moist adiabatic*.) On an aerological diagram, θ_e is the value of θ at which the pseudo-adiabat becomes parallel to the dry adiabat. This occurs when p is sufficiently low that r_s becomes zero (Fig. 1.6).

The mathematical expression of the thermodynamical processes outlined above is discussed now. Suppose the air is initially saturated. The rate of heating due to the condensation of water vapour is $\dot{q} = -(L/T)(dr_s/dt)$. In this case, the First Law of Thermodynamics (Eq. 1.2) becomes,

$$c_p \frac{d}{dt} \ln \theta = -\frac{L}{T} \frac{dr_s}{dt}.$$

In the spirit of the chapter, where only the simplest forms of the theory of used, we assume that the rate of change of r_s is much larger than the change in T or L. Integrating from the thermodynamic state (r_s, θ) to the state $(0, \theta_e)$ gives

$$\theta_e \approx \theta \exp\left(\frac{Lr_s}{c_p T}\right) \approx \theta \left(1 + \frac{Lr_s}{c_p T}\right).$$

(The assumption that the changes in T or L are small are not really justifiable and can be relaxed at the expense of additional mathematical complexity.)

θ_e is also conserved for unsaturated air. As the air is unsaturated, the relevant pseudo-adiabat θ_e must be found by lifting the parcel to the *saturation temperature* T_{LCL}. In this case,

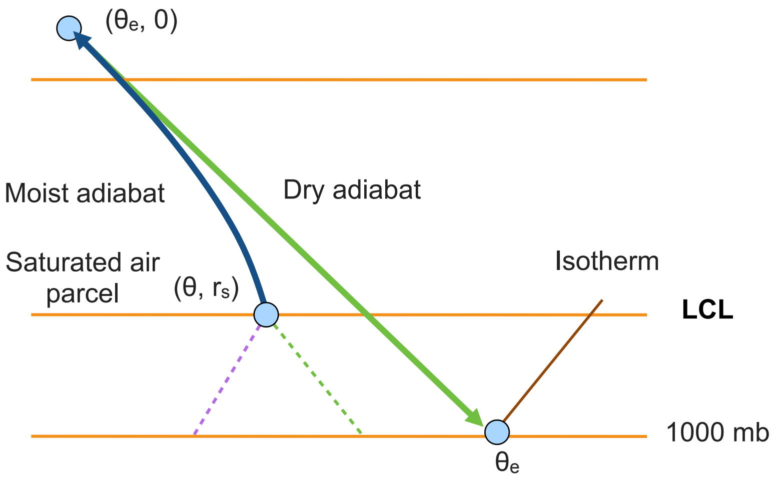

Figure 1.6: The physical interpretation of equivalent potential temperature θ_e and its calculation on an aerological diagram. The horizontal orange lines represent isobars. A saturated air parcel at p with thermodynamic properties (θ, r_s) is lifted to the point where $r_s = 0$. This thermodynamic process follows the thick blue line. The final value of θ is labeled θ_e. The value of θ_e is found by displacing the parcel downward to 1000 mb. As the parcel is now dry, the process is adiabatic and follows the green line, which is a dry adiabat. The temperature at 1000 mb is the value assigned to θ_e of the original saturated air parcel at p. If the air is unsaturated, it must be first lifted to saturation. This process is represented by the dashed lines, and the pressure at which saturation occurs is the lifting condensation level (LCL). In this way θ_e is conserved for both saturated and unsaturated air parcels.

$$\theta_e \approx \theta \exp\left(\frac{Lr}{c_p T_{TCL}}\right) \approx \theta\left(1 + \frac{Lr}{c_p T_{LCL}}\right).$$

Using the simplified expressions derived above to calculate the equivalent potential temperature can give relatively large errors (up to 3 K according to (Bolton, 1980)). An accurate, practical formula for the computation of equivalent potential temperature can be found in Bolton (1980).

1.6.4 Saturated equivalent potential temperature

If the environment is unsaturated, a hypothetically saturated version of θ_e using the temperature and pressure of the unsaturated parcel, can be calculated as

$$\theta_{es} \approx \theta \exp\left(\frac{Lr_s(T,p)}{c_p T_{LCL}}\right).$$

Here $r_s(T,p)$ means the saturated mixing ratio calculated with the temperature and pressure of the unsaturated parcel. When the environment is saturated, θ_{es} and θ_e the same. θ_{es} is useful in determining whether or not the atmosphere is conditionally unstable as it can be shown that the atmosphere is conditionally unstable if $d\,\theta_{es}/dz < 0$.

1.7 Convective Potential Energies

In section 1.4, Newton's Second Law was applied to a parcel of air and an expression relating the vertical acceleration of the parcel to the buoyancy force found (Eq. 1.6). In subsequent sections, the static instability and conditional instability were investigated from the framework of forces and accelerations. In the present section, moist convection and the development of downdraughts will be investigated from an energy perspective.

Equation 1.6 can be transformed into an equation for the parcel kinetic energy by first multiplying by w_{par} and then integrating from the initial time t_1, corresponding to height z_1 and pressure p_1, to some later time t_2, corresponding to height z_2 and pressure p_2. Whereupon,

$$\frac{1}{2}w_{par\,2}^2 - \frac{1}{2}w_{par\,1}^2 = \int_{z_1}^{z_2} b\,dz = R_d \int_{p_1}^{p_2} (T - T_{env})\,d\ln p. \tag{1.12}$$

The second equality is obtained using hydrostatic balance and the perfect gas law.

1.7.1 Convective Inhibition (CIN)

That work must be done in lifting a parcel from the surface to the LFC is a barrier to moist convection, preventing the instability from being released spontaneously. The energy need to surmount this barrier can be quantified as follows. Taking p_1 to be the surface (1000 mb) and p_2 be the LFC, the right hand side of Eq. 1.12 defines the *Convective Inhibition* (CIN). That is

$$CIN = R_d \int_{p_{1000}}^{p_{LFC}} (T - T_{env})\, d\ln p.$$

When, on an aerological diagram, the parcel temperature lies to the left of the environmental temperature, the area between the two curves is defined to be negative. The negative area is proportional to the CIN.

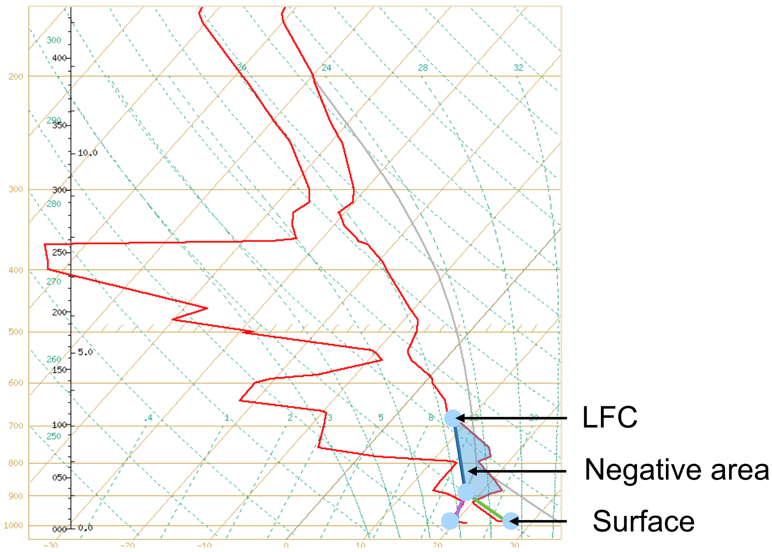

Figure 1.7: The Convective Inhibition (CIN) is the work required to lift a near-surface parcel to its LFC. As the parcel is negatively buoyant, the difference between the parcel and environmental temperatures is negative. On an aerological diagram the CIN is proportional to the area between the parcel and the environmental temperatures, which is negative.

1.7.2 Convective Available Potential Energy (CAPE)

In Eq. 1.12, let p_1 be the LFC and p_2 be the LNB. Then

$$CAPE = R_d \int_{p_{LFC}}^{p_{LNB}} (T - T_{env})\, d\ln p$$

is the *Convective Available Potential Energy* (CAPE) and $w_{par}(p_{LNB}) = \sqrt{2CAPE}$. The CAPE is the (maximum) potential energy in the environment that can be converted to kinetic energy by a conditionally unstable air parcel. On an aerological diagram, the CAPE is proportional to the area between the parcel temperature and the environmental temperature (Fig. 1.8).

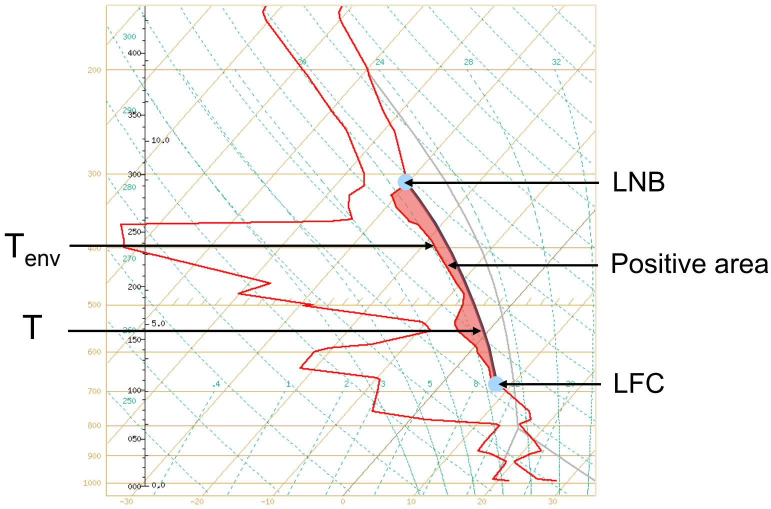

Figure 1.8: The Convective Available Potential Energy (CAPE) is the potential energy in the environment that can be converted to kinetic energy by a conditionally unstable air parcel. As the parcel is positively buoyant between the LFC and LNB, the difference between the parcel and environmental temperatures on this interval is positive. On an aerological diagram the CAPE is proportional to the area between the parcel and the environmental temperatures, which is positive.

1.7.3 Wet-Bulb Potential Temperature and Downdraught Convective Available Potential Energy (DCAPE)

As a convective cloud grows and precipitates, the evaporation of liquid water or the melting of ice becomes important as these thermodynamic processes often produce downdraughts and cold pools (for example, recall Figure 1.1b, c). This thermodynamic process can be described using the *wet-bulb potential temperature* θ_w.

The *wet-bulb temperature* T_w is the temperature to which an unsaturated parcel of air is cooled by evaporating water into it at constant pressure until it is saturated. It can be calculated on an aerological diagram through the following thought experiment. The unsaturated parcel has pressure p_i and is lifted dry adiabatically until saturation, or in other words, until it reaches the LCL. It then descends along a moist adiabat to p_i as water is constantly added to maintain saturation. T_w is the temperature of a saturated parcel once it has returned its original pressure p_i. The thought experiment is illustrated in Fig. 1.9. The *wet-bulb potential temperature* θ_w can be determined on an aerological diagram by first finding T_w and then displacing the parcel along a pseudo-adiabat to 1000 mb. See Davies-Jones (2008) for a more detailed discussion of wet-bulb potential temperature and practical methods for its calculation.

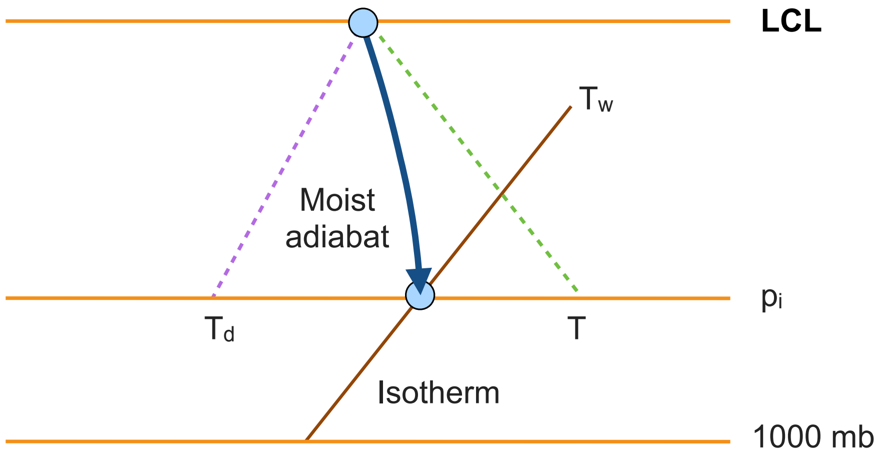

Figure 1.9: The calculation of the wet-bulb temperature T_w on an aerological diagram. An unsaturated parcel is lifted dry adiabatically from pressure p_i to the LCL. It then returns to p_i along a moist-adiabat with being water constantly added to maintain saturation. T_w is the temperature of the saturated parcel once it has returned its original pressure p_i. The horizontal orange lines represent isobars, the brown line an isotherm, the dashed green line an isentrope and the dashed violet line an isopleth of saturated mixing ratio.

Downdraughts are important parts of precipitating convection. They are driven by the negative buoyancy associated with the cooling that occurs when precipitation evaporates in unsaturated air or when it melts at the freezing level (and also in part by the drag exerted on the air by precipitation which is not discussed further here). The *Downdraught Convective Available Potential Energy* (DCAPE) is defined by the expression

$$DCAPE = R_d \int_{p_{1000}}^{p_i} (T - T_{env}) \, d \ln p,$$

which comes from Eq. 1.12 with p_1 replaced by p_{1000} and p_2 replaced by p_i, the assumed origin of the downdraught. In the calculation of DCAPE, the parcel follows a moist adiabat to 1000 mb (Fig. 1.10).

1.8 Summary

Most weather systems in the tropics depend critically on moist convection. Many of the most important examples will be discussed in this book; they include mesoscale convective systems, tropical cyclones, tropical waves, the Madden-Julian Oscillation,

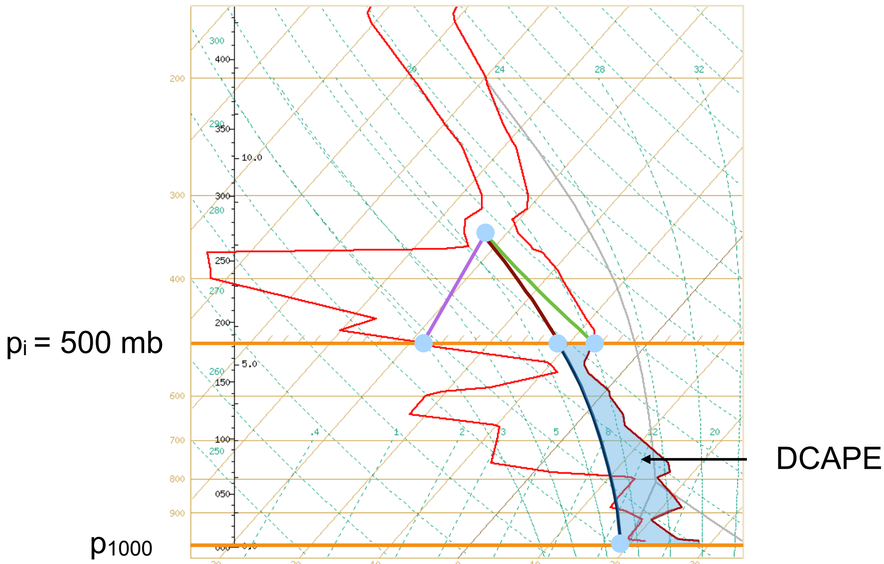

Figure 1.10: Downdraught Convective Available Potential Energy (DCAPE) is the potential energy generated when precipitation evaporates in unsaturated air or when it melts at the freezing level. This potential energy is then converted to kinetic energy, forming the downdraught. In this figure, it is assumed that the downdraught originates at $p_i = 500$ mb. On an aerological diagram the DCAPE is proportional to the area between the parcel and the environmental temperatures, which is positive.

the Hadley and Walker circulations. These systems are tightly coupled to the heating provided by deep convection. And for this reason, variations in the frequency, structure or intensity of weather systems exert a strong control on the variability of rainfall.

The present chapter reviews some of the main ideas on dry convective instability (static instability) and moist convective instability (conditional instability), and sets the scene for much of the material in the subsequent chapters.

The instability of a lifted parcel is checked and the development of convection forecast by calculating its buoyancy. Typically, the parcel is initially unsaturated and stable to vertical displacements. However, bringing the parcel to saturation through further ascent can make it unstable to further displacements. This kind of instability is called conditional instability, and is highly relevant to the initiation of moist convection. Another important thermodynamic process relevant to moist convection is the evaporation of water into an unsaturated atmosphere. This process produces negative buoyancy and is responsible for the generation of downdraughts and cold pools. Moreover, the processes leading to conditional instability or downdraughts and cold pools can be simply depicted on an aerological diagram.

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