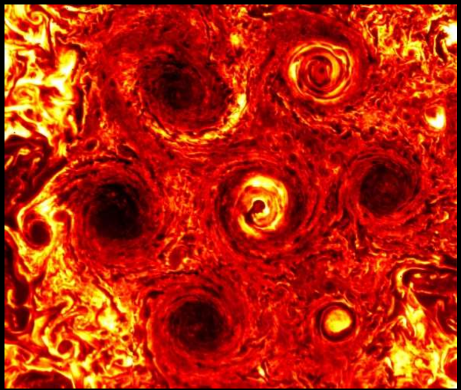
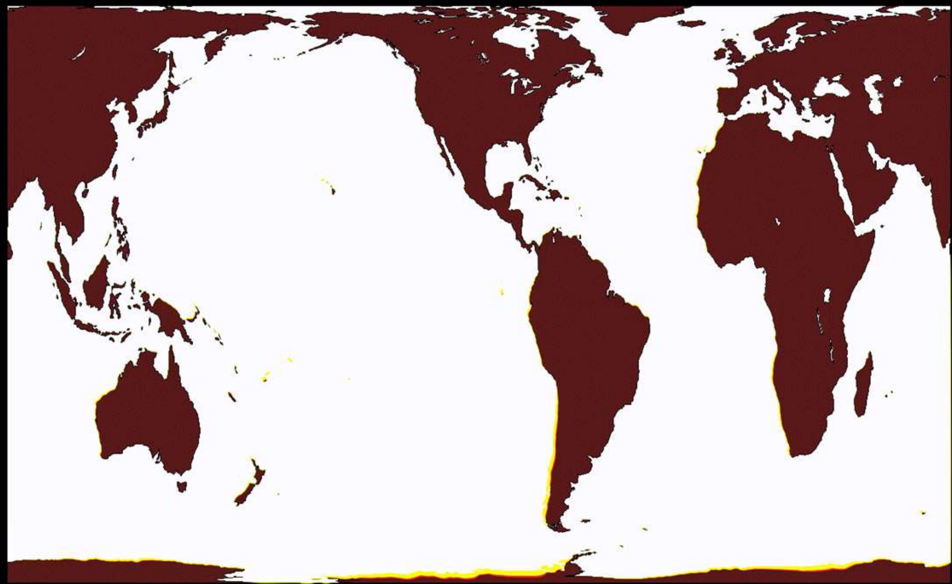
Lecture 7:

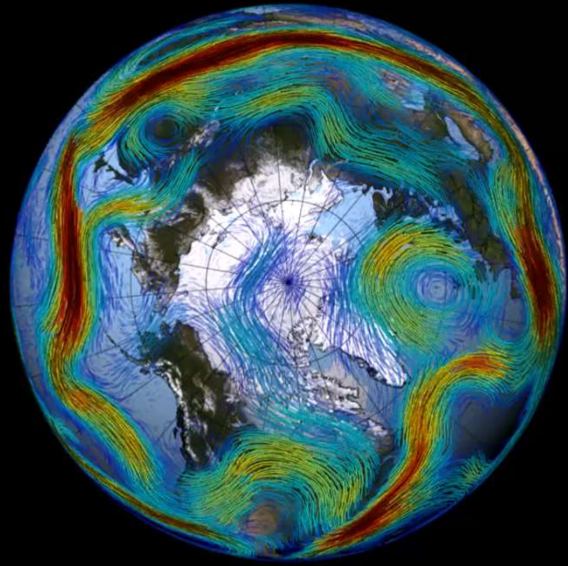
Potential vorticity, Rossby waves and internal waves

Callum J. Shakespeare

Fellow, Climate and Fluid Physics, ANU







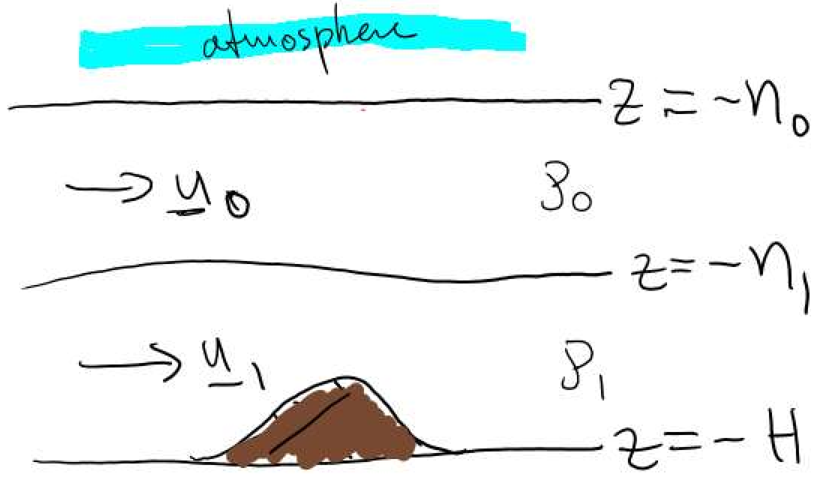
2-layer SWE from Lecture 6

$$\frac{Du_0}{Dt} - f v_0 = g\frac{\partial \eta_0}{\partial x} \qquad \frac{Du_1}{Dt} - f v_1 = g\frac{\partial \eta_0}{\partial x} + g'\frac{\partial \eta_1}{\partial x}$$

$$\frac{Dv_0}{Dt} + f u_0 = g\frac{\partial \eta_0}{\partial y} \qquad \frac{Dv_1}{Dt} + f u_1 = g\frac{\partial \eta_0}{\partial y} + g'\frac{\partial \eta_1}{\partial y}$$

$$\frac{Dh_0}{Dt} = -h_0 \nabla \cdot \mathbf{u_0} \qquad \frac{Dh_1}{Dt} = -h_1 \nabla \cdot \mathbf{u_1}$$

$$h_0 = \eta_1 - \eta_0 \qquad h_1 = H - \eta_1$$



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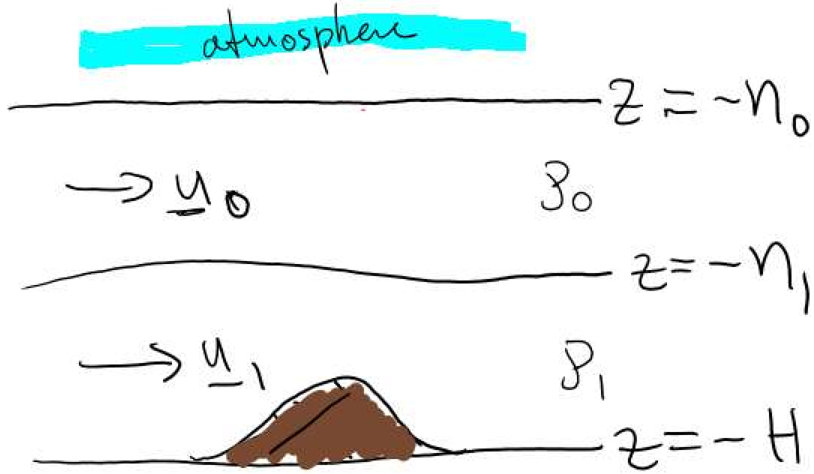
$$-\frac{\partial}{\partial y}\ \frac{Du_1}{Dt} - fv_1 = g\frac{\partial \eta_0}{\partial x} + g'\frac{\partial \eta_1}{\partial x}$$

$$\frac{\partial}{\partial x}\ \frac{Dv_1}{Dt} + fu_1 = g\frac{\partial \eta_0}{\partial y} + g'\frac{\partial \eta_1}{\partial y}$$

$$\frac{Dh_1}{Dt} = -h_1 \nabla \cdot \mathbf{u_1}$$

$$h_1 = H - \eta_1$$



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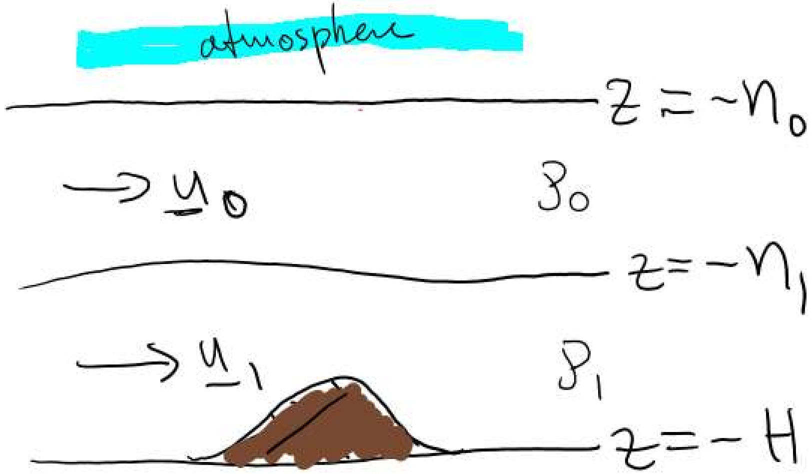

$$\frac{\partial}{\partial x}\left(\frac{\partial v_1}{\partial t} + \underline{u}_1\cdot\nabla v_1 + f u_1\right) - \frac{\partial}{\partial y}\left(\frac{\partial u_1}{\partial t} + \underline{u}_1\cdot\nabla u_1 - f v_1\right) = 0.$$

$$\Rightarrow \left(\frac{\partial}{\partial t} + \underline{u}_1\cdot\nabla\right)\left(\frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y}\right) + \frac{\partial \underline{u}_1}{\partial x}\cdot\nabla v_1 - \frac{\partial \underline{u}_1}{\partial y}\cdot\nabla u_1$$

$$+ f\left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y}\right) + v\,\frac{\partial f}{\partial y} = 0.$$

2-layer SWE from Lecture 6

$$-\frac{\partial}{\partial y}\;\frac{Du_1}{Dt} - f v_1 = \cancel{g\frac{\partial \eta_0}{\partial x} + g'\frac{\partial \eta_1}{\partial x}}$$

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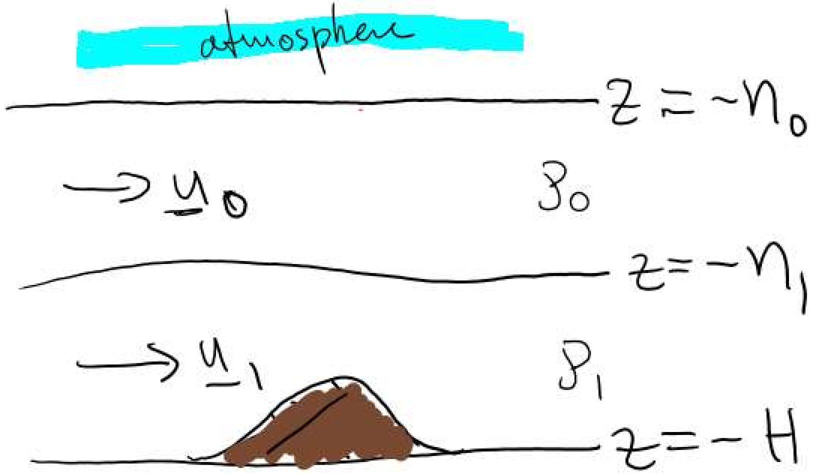

$$\frac{\partial}{\partial x}\left(\frac{\partial v_1}{\partial t} + \underline{u}_1\cdot\nabla v_1 + f u_1\right) - \frac{\partial}{\partial y}\left(\frac{\partial u_1}{\partial t} + \underline{u}_1\cdot\nabla u_1 - f v_1\right) = 0.$$

$$\Rightarrow \underbrace{\left(\frac{\partial}{\partial t} + \underline{u}_1\cdot\nabla\right)}_{\frac{D}{Dt}}\underbrace{\left(\frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y}\right)}_{\omega_1} + \underbrace{\frac{\partial \underline{u}_1}{\partial x}\cdot\nabla v_1 - \frac{\partial \underline{u}_1}{\partial y}\cdot\nabla u_1}$$

$$+ f\underbrace{\left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y}\right)}_{\nabla\cdot\underline{u}_1 = -\frac{1}{h_1}\frac{Dh_1}{Dt}} + v\frac{\partial f}{\partial y} = 0.$$

2-layer SWE from Lecture 6

$$-\frac{\partial}{\partial y}\quad \frac{Du_1}{Dt} - f v_1 = \cancel{g\frac{\partial \eta_0}{\partial x} + g'\frac{\partial \eta_1}{\partial x}}$$

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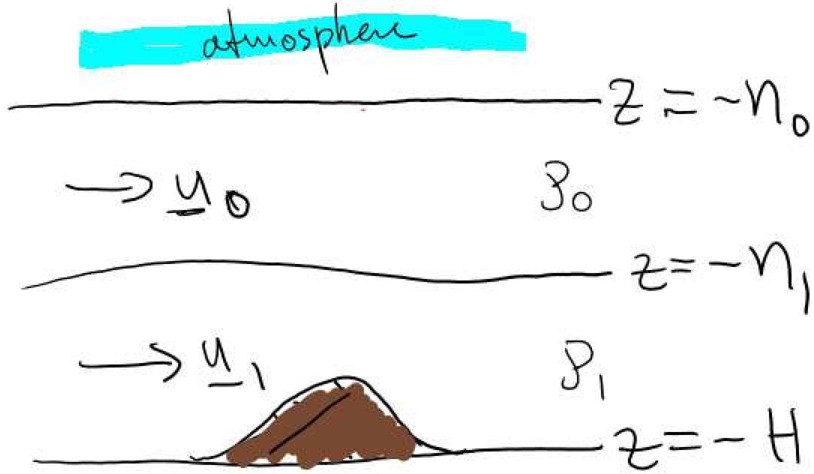

$$\frac{\partial}{\partial x}\left(\frac{\partial v_1}{\partial t} + \underline{u}_1\cdot\nabla v_1 + f u_1\right) - \frac{\partial}{\partial y}\left(\frac{\partial u_1}{\partial t} + \underline{u}_1\cdot\nabla u_1 - f v_1\right) = 0.$$

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$$\frac{\partial u_1}{\partial x}\frac{\partial v_1}{\partial x} + \frac{\partial v_1}{\partial x}\frac{\partial v_1}{\partial y} - \frac{\partial u_1}{\partial y}\frac{\partial u_1}{\partial x} - \frac{\partial v_1}{\partial y}\frac{\partial u_1}{\partial y}$$

$$= \frac{\partial u_1}{\partial x}\left(\frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y}\right) + \frac{\partial v_1}{\partial y}\left(\frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y}\right)$$

$$= \omega_1 \nabla\cdot\underline{u}_1$$

2-layer SWE from Lecture 6

$$-\frac{\partial}{\partial y}\quad \frac{Du_1}{Dt} - f v_1 = \cancel{g\frac{\partial \eta_0}{\partial x} + g'\frac{\partial \eta_1}{\partial x}}$$

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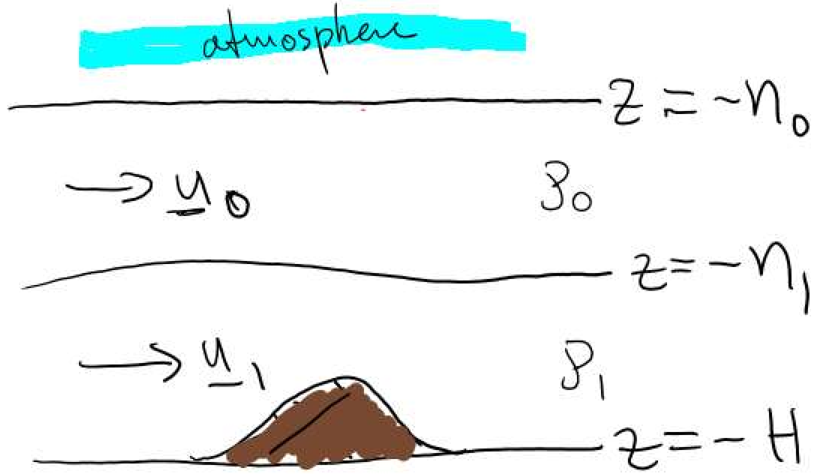

$$\frac{D\omega_1}{Dt} + \omega_1 \nabla\cdot\underline{u}_1 + f\,\nabla\cdot\underline{u}_1 + v\frac{\partial f}{\partial y} = 0$$

$$\Rightarrow \frac{D}{Dt}(f+\omega_1) + (\omega_1+f)\left(-\frac{1}{h_1}\frac{Dh_1}{Dt}\right) = 0$$

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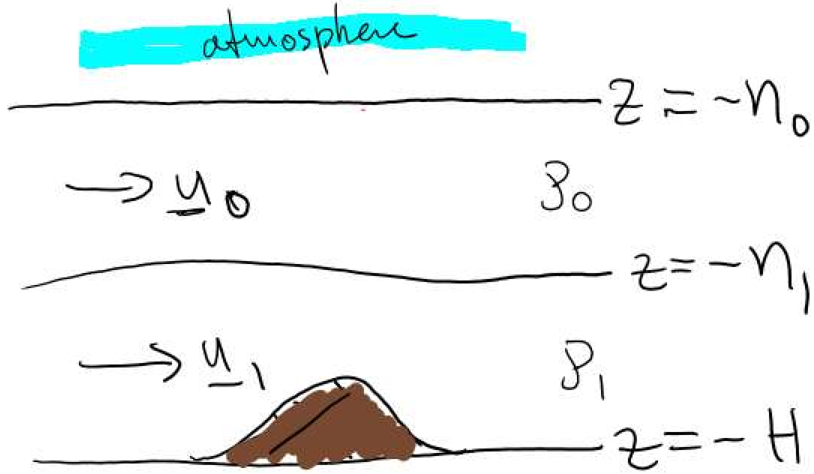

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The quantity

$$q_1 = \frac{\omega_1 + f}{h_1} = \frac{\omega_1 + f}{H - \eta_1}$$

is conserved

This is the **Potential Vorticity**

2-layer SWE from Lecture 6

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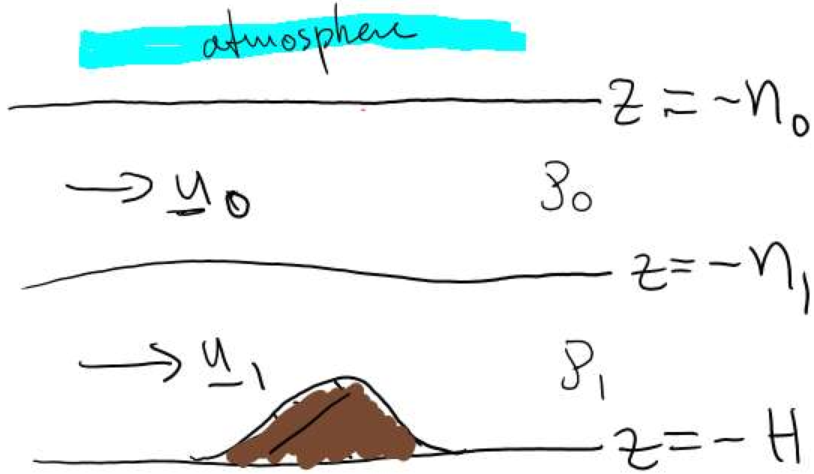

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Same reasoning for layer 0:

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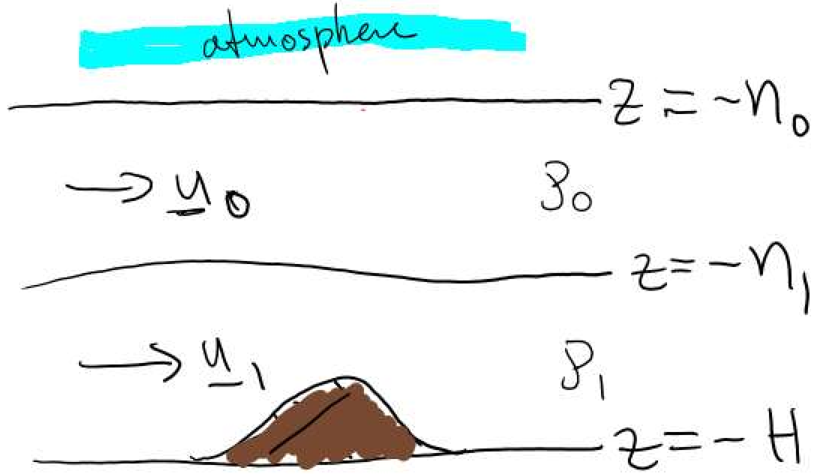

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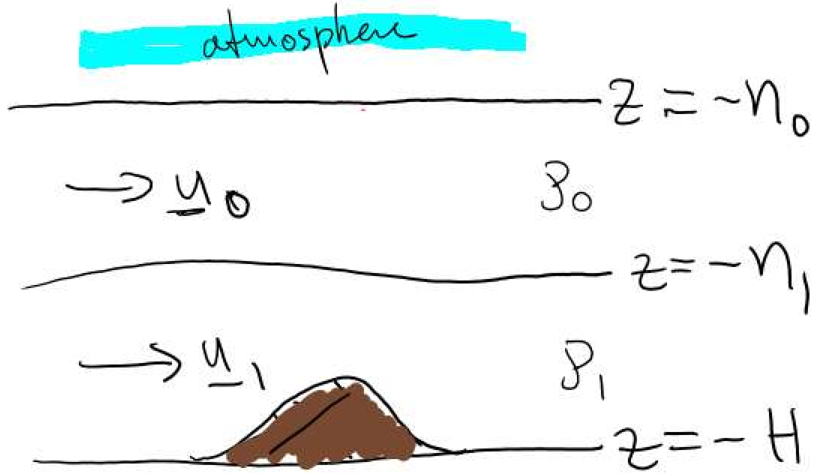
PV Conservation

Suppose that: $\omega \ll f$ and $\eta_i \ll H$ and the flow is STEADY

Then: $q_1 \simeq \dfrac{f}{H}$

And: $\dfrac{Dq_1}{Dt} = \vec{u} \cdot \nabla q_1 = \vec{u} \cdot \nabla \dfrac{f}{H} = 0$



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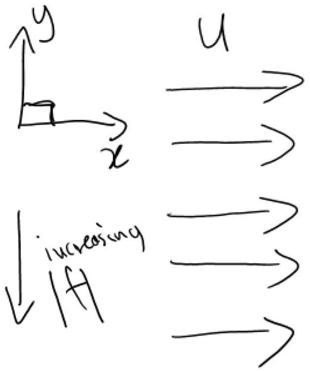
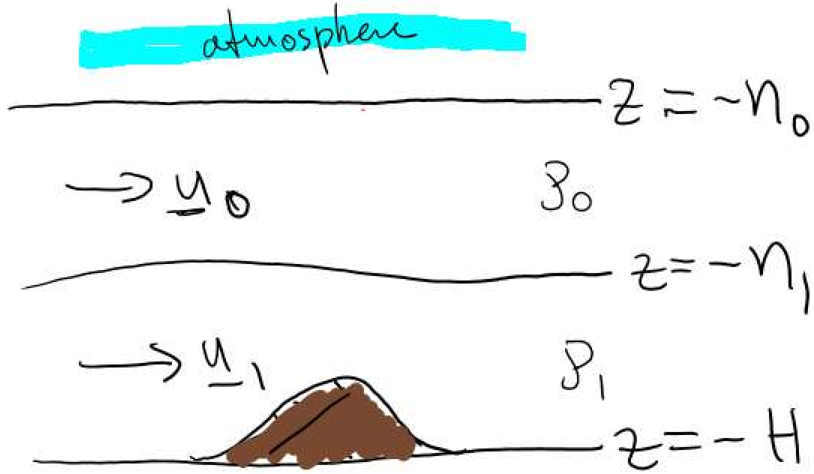
The flow follows contours of f/H



Which hemisphere is this in?



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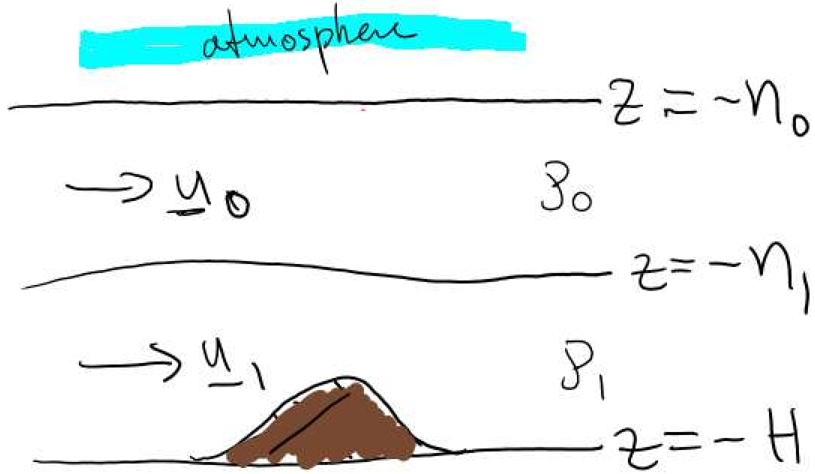

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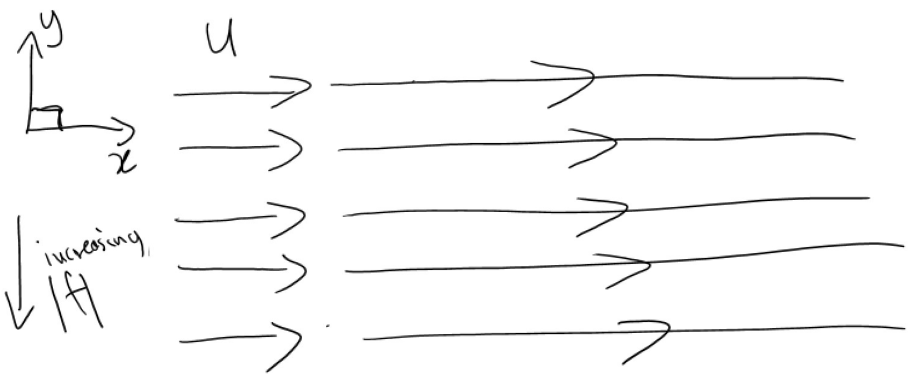

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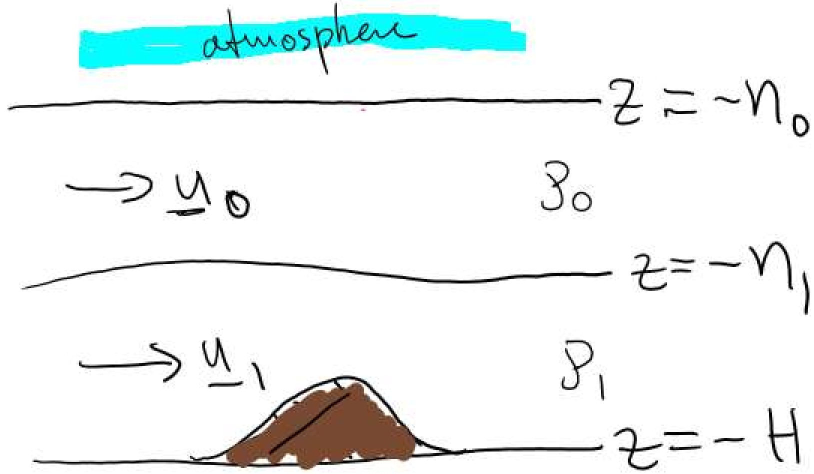

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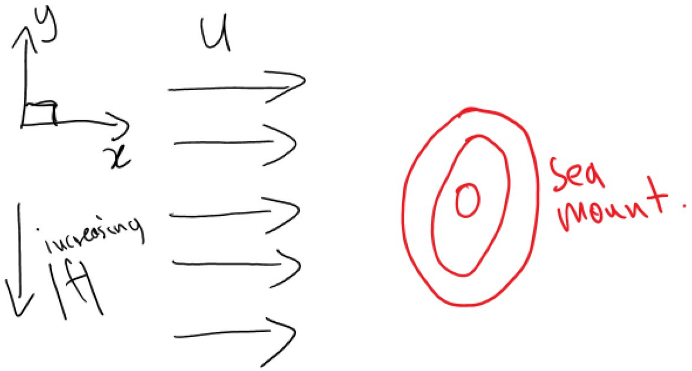

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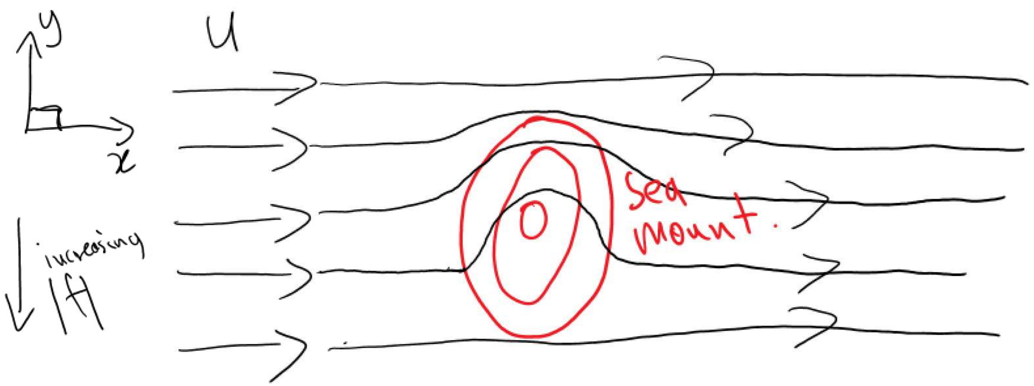
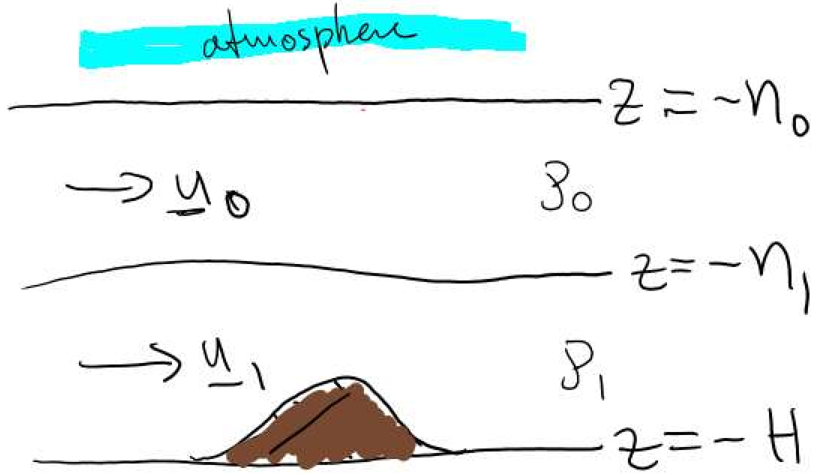
The flow follows contours of f/H
If H gets smaller, so must |f|….





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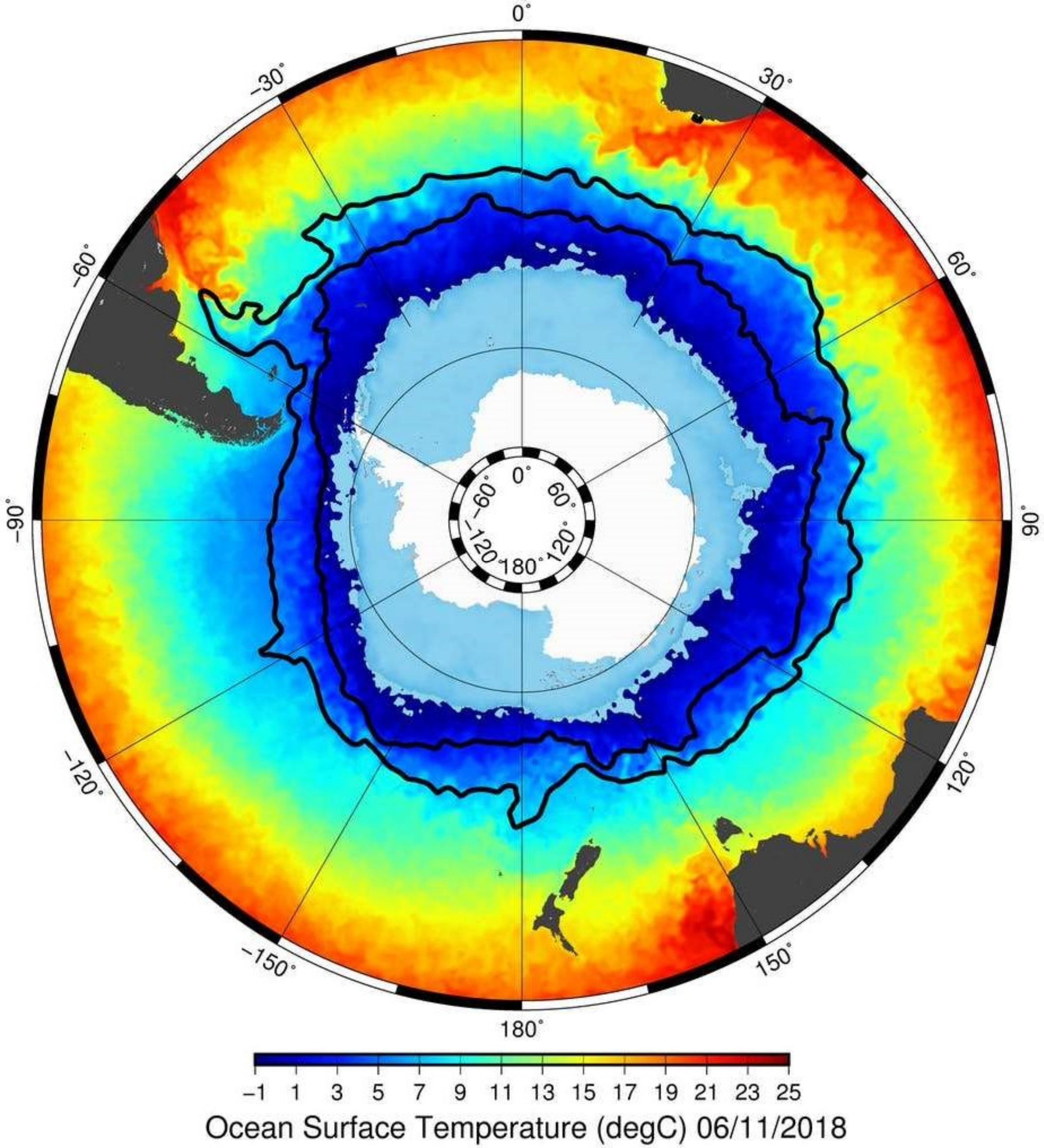
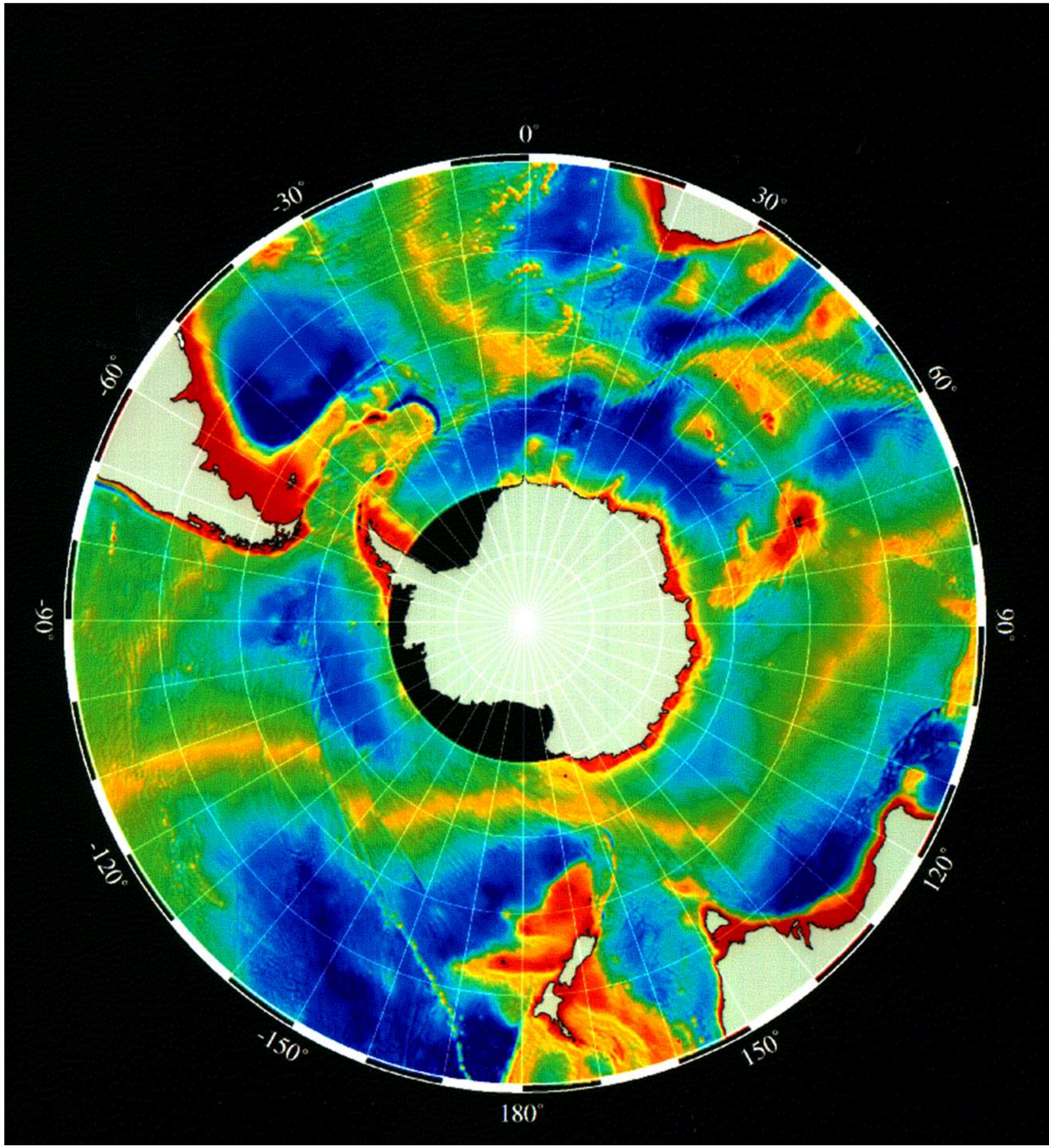


0°
30°
60°
90°
120°
150°
180°
−150°
−120°
−90°
−60°
−30°
0°
60°
120°
180°
−120°
−60°
−1 1 3 5 7 9 11 13 15 17 19 21 23 25
Ocean Surface Temperature (degC) 06/11/2018



0°
30°
60°
90°
120°
150°
180°
-150°
-120°
-90°
-60°
-30°

New 2 layer SW Equations

X momentum

$$\frac{Du_0}{Dt} - fv_0 = g\frac{\partial \eta_0}{\partial x} \qquad \frac{Du_1}{Dt} - fv_1 = g\frac{\partial \eta_0}{\partial x} + g'\frac{\partial \eta_1}{\partial x}$$

Y momentum

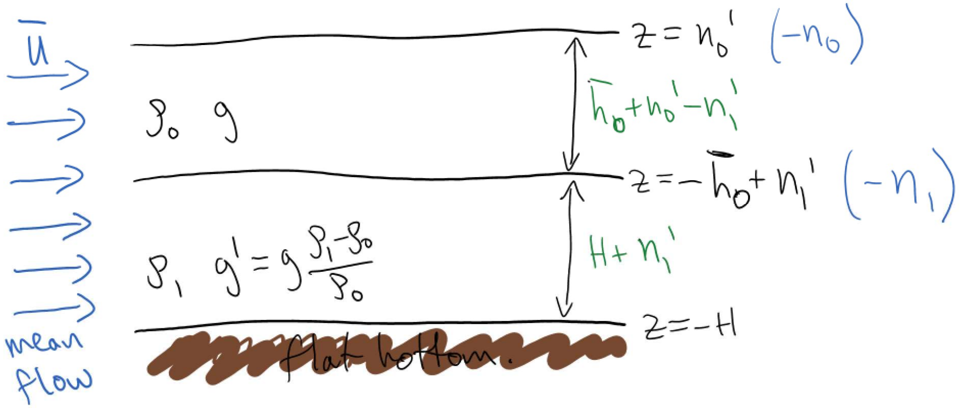
$$\frac{Dv_0}{Dt} + fu_0 = g\frac{\partial \eta_0}{\partial y} \qquad \frac{Dv_1}{Dt} + fu_1 = g\frac{\partial \eta_0}{\partial y} + g'\frac{\partial \eta_1}{\partial y}$$

Conserve PV

$$\frac{D}{Dt}\left(\frac{\omega_0 + f}{h_0}\right) = 0 \qquad \frac{D}{Dt}\left(\frac{\omega_1 + f}{h_1}\right) = 0$$

$$h_0 = \eta_1 - \eta_0 \qquad h_1 = H - \eta_1$$

Linearisation



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$$\frac{D}{Dt}\left(\frac{\omega_0 + f}{h_0}\right) = 0$$
$$h_0 = \eta_1 - \eta_0$$

$$\frac{Du_1}{Dt} - fv_1 = g\frac{\partial \eta_0}{\partial x} + g'\frac{\partial \eta_1}{\partial x}$$
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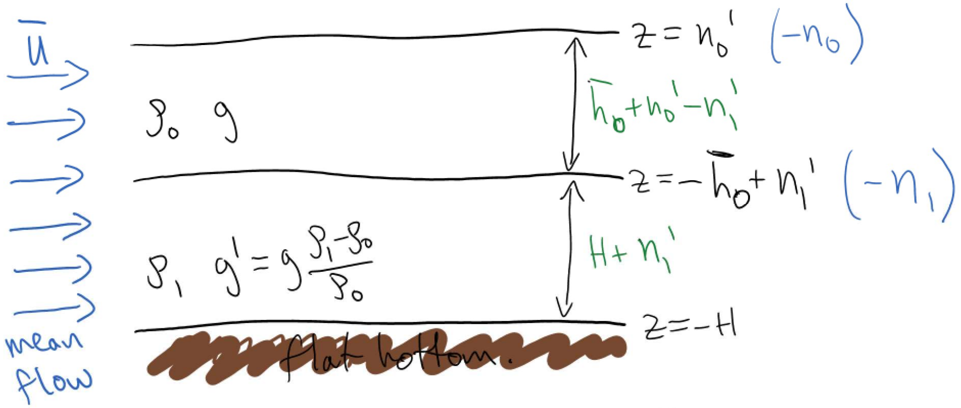

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$$\boxed{\frac{D}{Dt}\left(\frac{\omega_0+f}{h_0}\right) = 0} \qquad \frac{D}{Dt}\left(\frac{\omega_1+f}{h_1}\right) = 0$$

$$h_0 = \eta_1 - \eta_0 \qquad h_1 = H - \eta_1$$

Assume: $f = f_0 + \beta y \qquad \beta y \ll f_0$

$\bar{u} \gg u \qquad \eta' \ll \bar{h}_0, H$

$$\left(\frac{\partial}{\partial t} + (u_0+\bar{u})\frac{\partial}{\partial x} + v_0\frac{\partial}{\partial y}\right)\left(\frac{\omega_0+f_0+\beta y}{h_0}\right) = 0$$

Linearisation

$$\frac{Du_0}{Dt} - f v_0 = g\frac{\partial \eta_0}{\partial x}$$
$$\frac{Dv_0}{Dt} + f u_0 = g\frac{\partial \eta_0}{\partial y}$$
$$\boxed{\frac{D}{Dt}\left(\frac{\omega_0 + f}{h_0}\right) = 0}$$
$$h_0 = \eta_1 - \eta_0$$

$$\frac{Du_1}{Dt} - f v_1 = g\frac{\partial \eta_0}{\partial x} + g'\frac{\partial \eta_1}{\partial x}$$
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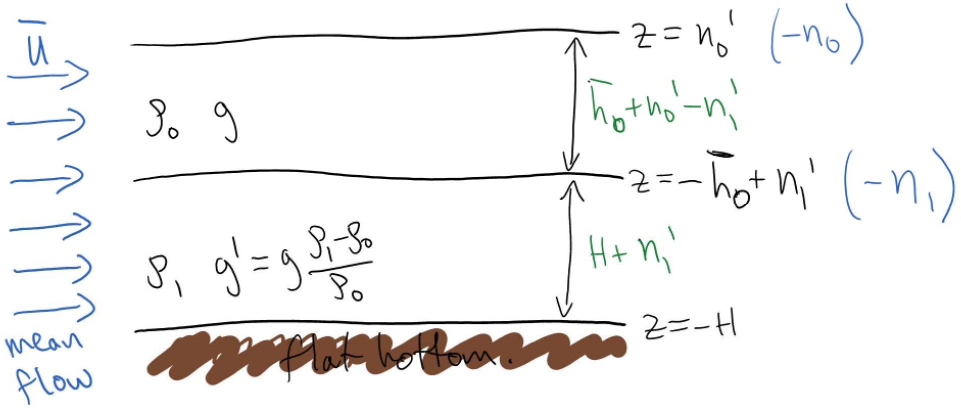
$$\frac{D}{Dt}\left(\frac{\omega_1 + f}{h_1}\right) = 0$$
$$h_1 = H - \eta_1$$



Assume: $f = f_0 + \beta y \qquad \beta y \ll f_0$

$\bar{u} \gg u \qquad \eta' \ll \bar{h}_0, H$

$$\left(\frac{\partial}{\partial t} + (u_0 + \bar{u})\frac{\partial}{\partial x} + v_0\frac{\partial}{\partial y}\right)\left(\frac{\omega_0 + f_0 + \beta y}{h_0}\right) = 0$$

$$h_0 \approx \bar{h}_0 + \eta_0' - \eta_1'$$

$$\frac{1}{h_0} = \frac{1}{\bar{h}_0\left(1 + \frac{\eta_0' - \eta_1'}{\bar{h}_0}\right)}$$

$$\approx \frac{1}{\bar{h}_0}\left(1 - \frac{\eta_0' - \eta_1'}{\bar{h}_0}\right)$$

$$\Rightarrow \left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\left(\omega_0 - \frac{\eta_0' - \eta_1'}{\bar{h}_0} f_0\right) + v_0 \beta = 0$$

Linearisation

$$\boxed{\frac{Du_0}{Dt} - f v_0 = g\frac{\partial \eta_0}{\partial x}}$$

$$\frac{Dv_0}{Dt} + f u_0 = g\frac{\partial \eta_0}{\partial y}$$

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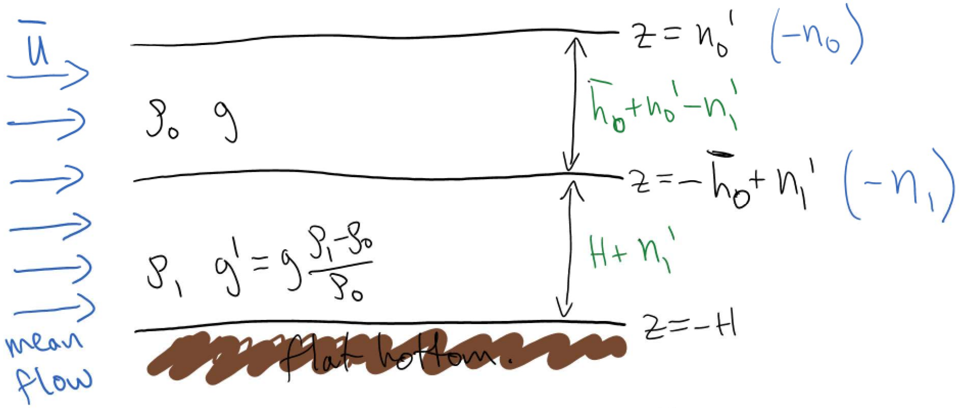


Assume: $f = f_0 + \beta y \qquad \beta y \ll f_0$

$\bar{U} \gg u \qquad \eta' \ll \bar{h}_0, H$

$$\left(\frac{\partial}{\partial t} + \left(\cancel{u_0} + \bar{U}\right)\frac{\partial}{\partial x} + \cancel{v_0\frac{\partial}{\partial y}}\right)u_0 - \left(f_0 + \cancel{\beta y}\right)v_0 = -g\frac{\partial \eta_0'}{\partial x}$$

$$\Rightarrow \left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)u_0 - f_0 v_0 = -g\frac{\partial \eta_0'}{\partial x}$$

$$\left(\frac{\partial}{\partial t} + (u_0 + \bar{U})\frac{\partial}{\partial x} + v_0\frac{\partial}{\partial y}\right)\underbrace{\left(\frac{\omega_0 + f_0 + \beta y}{h_0}\right)}_{} = 0$$

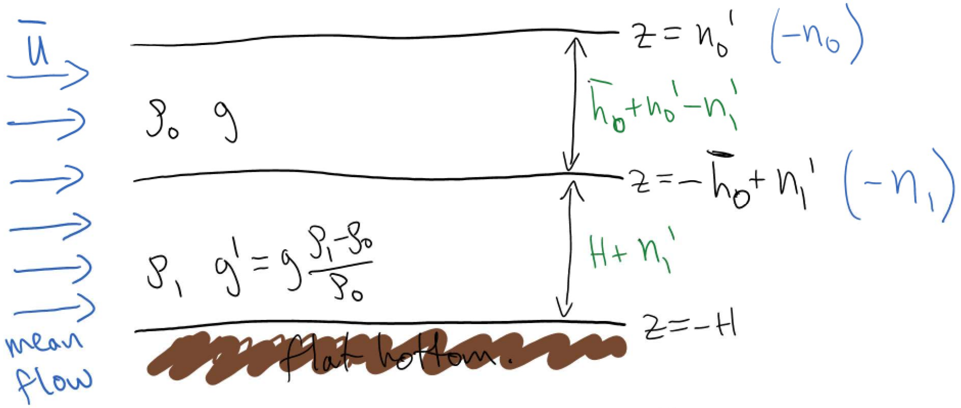
$$h_0 \approx \bar{h}_0 + \eta_0' - \eta_1'$$

$$\frac{1}{h_0} = \frac{1}{\bar{h}_0\left(1 + \frac{\eta_0' - \eta_1'}{\bar{h}_0}\right)}$$

$$\approx \frac{1}{\bar{h}_0}\left(1 - \frac{\eta_0' - \eta_1'}{\bar{h}_0}\right)$$

$$\Rightarrow \left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_0 - \frac{\eta_0' - \eta_1'}{\bar{h}_0}f_0\right) + v_0\beta = 0$$

Linearisation



$$\boxed{\frac{Du_0}{Dt}-fv_0=g\frac{\partial\eta_0}{\partial x}}$$

$$\frac{Dv_0}{Dt}+fu_0=g\frac{\partial\eta_0}{\partial y}$$

$$\frac{D}{Dt}\left(\frac{\omega_0+f}{h_0}\right)=0$$

$$h_0=\eta_1-\eta_0$$

$$\frac{Du_1}{Dt}-fv_1=g\frac{\partial\eta_0}{\partial x}+g'\frac{\partial\eta_1}{\partial x}$$

$$\frac{Dv_1}{Dt}+fu_1=g\frac{\partial\eta_0}{\partial y}+g'\frac{\partial\eta_1}{\partial y}$$

$$\frac{D}{Dt}\left(\frac{\omega_1+f}{h_1}\right)=0$$

$$h_1=H-\eta_1$$

$$\left(\frac{\partial}{\partial t}+(\cancel{u_0}+\bar{u})\frac{\partial}{\partial x}+\cancel{v_0\frac{\partial}{\partial y}}\right)u_0-(f_0+\cancel{\beta y})v_0=-g\frac{\partial\eta_0'}{\partial x}$$

$$\Rightarrow\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)u_0-f_0v_0=-g\frac{\partial\eta_0'}{\partial x}$$

Assume: $f=f_0+\beta y$ $\quad\beta y\ll f_0$

$\bar{u}\gg u$ $\quad\eta'\ll\bar{h}_0, H$

$$\left(\frac{\partial}{\partial t}+(u_0+\bar{u})\frac{\partial}{\partial x}+v_0\frac{\partial}{\partial y}\right)\left(\frac{\omega_0+f_0+\beta y}{h_0}\right)=0$$

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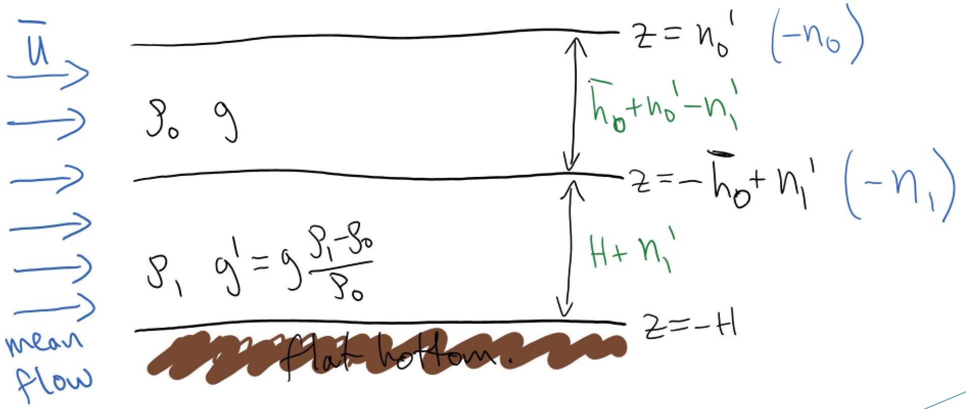
$$\frac{1}{h_0}=\frac{1}{\bar{h}_0\left(1+\frac{\eta_0'-\eta_1'}{\bar{h}_0}\right)}$$

$$\approx\frac{1}{\bar{h}_0}\left(1-\frac{\eta_0'-\eta_1'}{\bar{h}_0}\right)$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\left(\omega_0-\frac{\eta_0'-\eta_1'}{\bar{h}_0}f_0\right)+v_0\beta=0$$

Linearisation



$$\frac{Du_0}{Dt} - fv_0 = g\frac{\partial \eta_0}{\partial x}$$
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$$\frac{D}{Dt}\left(\frac{\omega_1 + f}{h_1}\right) = 0$$
$$h_1 = H - \eta_1$$

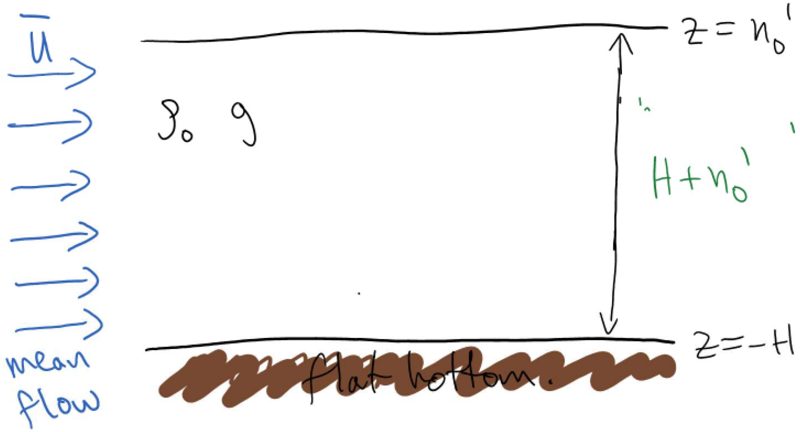
Linearised layer 0

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)u_0 - f_0 v_0 = -g\frac{\partial \eta_0'}{\partial x}$$
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)v_0 + f_0 u_0 = -g\frac{\partial \eta_0'}{\partial y}$$
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_0 - \frac{f_0}{h_0}(\eta_0' - \eta_1')\right) + \beta v_0 = 0$$

Linearised layer 1

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)u_1 - f_0 v_1 = -g\frac{\partial \eta_0'}{\partial x} - g'\frac{\partial \eta_1'}{\partial x}$$
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)v_1 + f_0 u_1 = -g\frac{\partial \eta_0'}{\partial y} - g'\frac{\partial \eta_1'}{\partial y}$$
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_1 - \frac{f_0}{h_1}\eta_1'\right) + \beta v_1 = 0$$

Rossby waves: barotropic

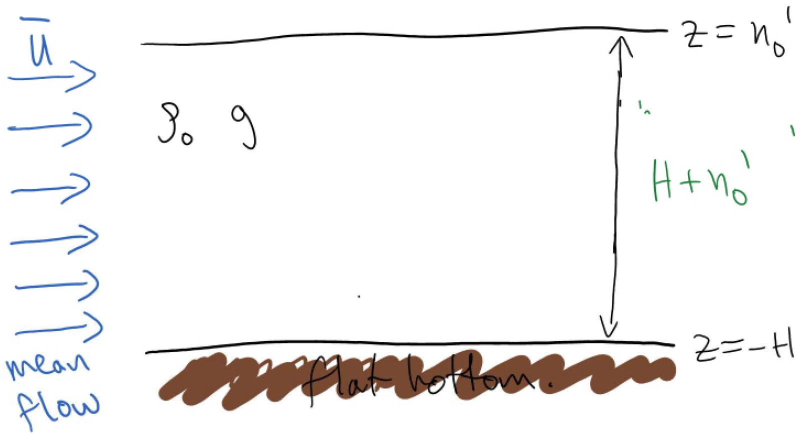


$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right) u_0 - f_0 v_0 = -g\frac{\partial \eta_0'}{\partial x}$$

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right) v_0 + f_0 u_0 = -g\frac{\partial \eta_0'}{\partial y}$$

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_0 - \frac{f_0}{H}\eta_0'\right) + \beta v_0 = 0$$

Rossby waves: barotropic



$\bar{U}\frac{\partial}{\partial x} << f \qquad \frac{\partial}{\partial t} << f$

No timescales of O(f) = no BGWs or other gravity waves

$$\cancel{\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right) u_0} - f_0 v_0 = -g\frac{\partial \eta_0'}{\partial x}$$

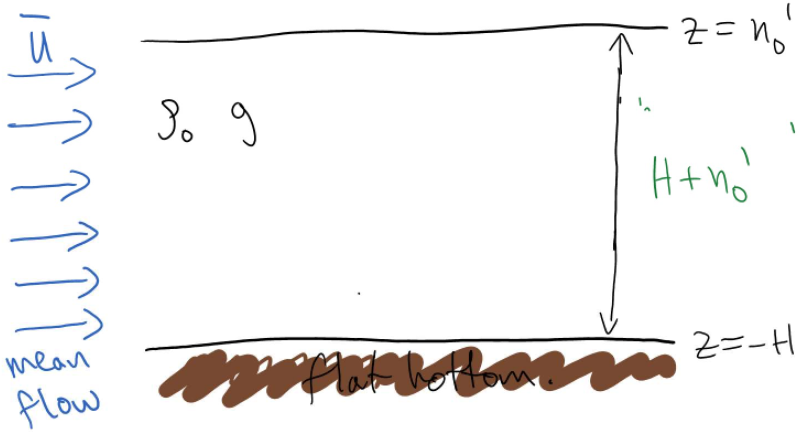
$$\cancel{\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right) v_0} + f_0 u_0 = -g\frac{\partial \eta_0'}{\partial y}$$

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_0 - \frac{f_0}{H}\eta_0'\right) + \beta v_0 = 0$$

Keep here as it's the same order as β

- This assumption means we are describing "balanced flow" since velocities are in geostrophic balance.
- State known as **quasi-geostrophic balance (QG)**
- There is a more general non-linear form which we won't look at here…

Rossby waves: barotropic



$\bar{U}\frac{\partial}{\partial x} \ll f \qquad \frac{\partial}{\partial t} \ll f$

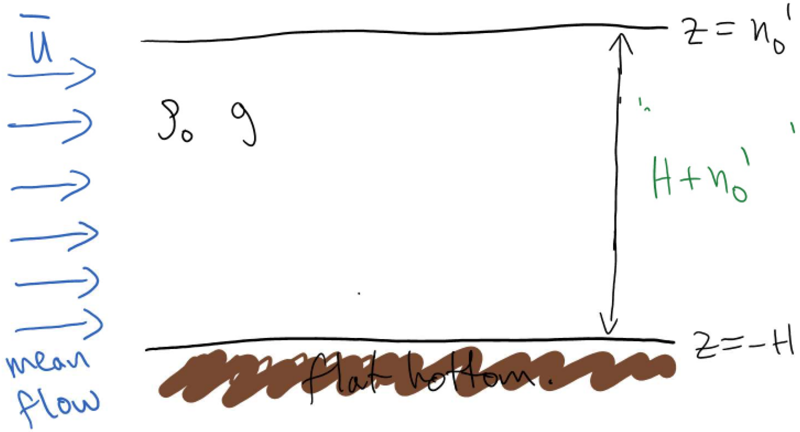
No timescales of O(f) = no BGWs or other gravity waves

$$\left.\begin{aligned}
\cancel{\left(\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right)} u_0 - f_0 v_0 &= -g\frac{\partial \eta_0'}{\partial x} \\
\cancel{\left(\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right)} v_0 + f_0 u_0 &= -g\frac{\partial \eta_0'}{\partial y}
\end{aligned}\right\} \quad \omega_0 = \frac{g}{f_0}\nabla^2\eta_0'$$

$$\left(\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right)\left(\omega_0 - \frac{f_0}{H}\eta_0'\right) + \beta v_0 = 0$$

Keep here as it's the same order as β

Rossby waves: barotropic



Linearised barotropic “quasi-geostrophy”

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\frac{g}{f_0}\nabla^2\eta_0' - \frac{f_0}{H}\eta_0'\right) + \frac{g\beta}{f_0}\frac{\partial \eta_0'}{\partial x} = 0$$

$\bar{U}\frac{\partial}{\partial x} \ll f \qquad \frac{\partial}{\partial t} \ll f$

No timescales of O(f) = no BGWs or other gravity waves

$$\cancel{\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)} u_0 - f_0 v_0 = -g\frac{\partial \eta_0'}{\partial x}$$

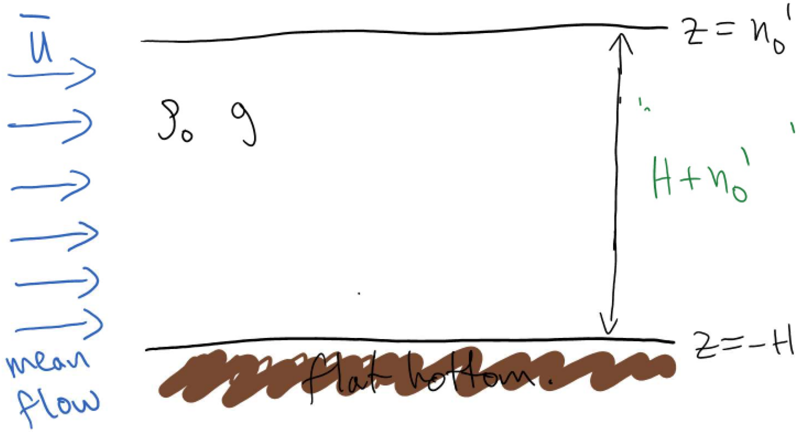
$$\cancel{\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)} v_0 + f_0 u_0 = -g\frac{\partial \eta_0'}{\partial y}$$

$$\omega_0 = \frac{g}{f_0}\nabla^2\eta_0'$$

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_0 - \frac{f_0}{H}\eta_0'\right) + \beta v_0 = 0$$

Keep here as it’s the same order as β

Rossby waves: barotropic



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Let $\eta_0' = \hat{\eta}_0 e^{i(kx+ly-\omega t)}$

$$\left[(-i\omega + ik\bar{u})\left(-K^2 - \frac{f_0^2}{gH}\right) + ik\beta\right]\hat{\eta}_0' = 0$$

$$\Rightarrow \quad \omega - k\bar{u} = \frac{-\beta k}{K^2 + \frac{f_0^2}{gH}}$$

$\bar{u}\frac{\partial}{\partial x} << f \qquad \frac{\partial}{\partial t} << f$

No timescales of O(f) = no BGW
or other gravity waves

$$\cancel{\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)} u_0 - f_0 v_0 = -g\frac{\partial \eta_0'}{\partial x}$$

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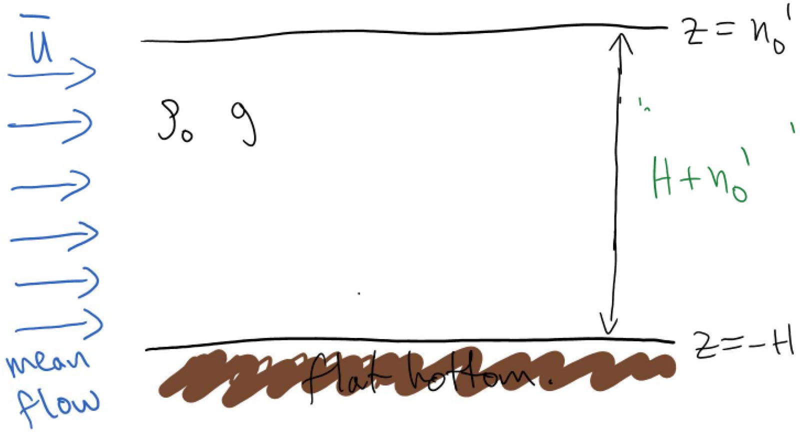
$\Big\}\ \omega_0 = \frac{g}{f_0}\nabla^2\eta_0'$

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_0 - \frac{f_0}{H}\eta_0'\right) + \beta v_0 = 0$$

Keep here as it's
the same order
as β

Rossby waves: barotropic



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Linearised barotropic "quasi-geostrophy"

$$\left(\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right)\left(\frac{g}{f_0}\nabla^2\eta_0' - \frac{f_0}{H}\eta_0'\right) + \frac{g\beta}{f_0}\frac{\partial \eta_0'}{\partial x} = 0$$

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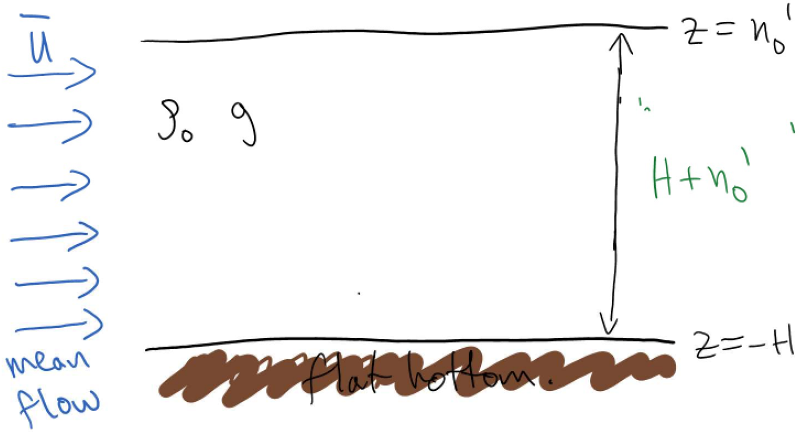
? ? ?

Rossby waves: barotropic



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Keep here as it's the same order as β

Linearised barotropic "quasi-geostrophy"

$$\left(\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right)\left(\frac{g}{f_0}\nabla^2\eta_0' - \frac{f_0}{H}\eta_0'\right) + \frac{g\beta}{f_0}\frac{\partial \eta_0'}{\partial x} = 0$$

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$$\Rightarrow \quad \omega - k\bar{u} = \frac{-\beta k}{K^2 + \frac{f_0^2}{gH}}$$

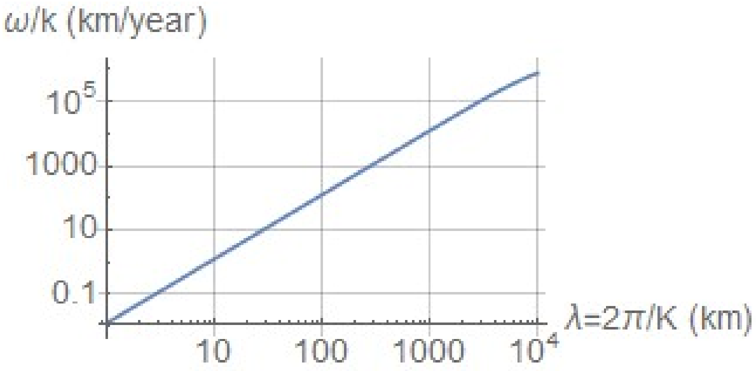
? ? ?

Rossby waves

- Doppler shifted by the mean flow
- Travel westward $\frac{\omega}{k} < 0$
- Depend on the Rossby radius $L_d = \frac{\sqrt{gH}}{f_0} \sim 2000km$

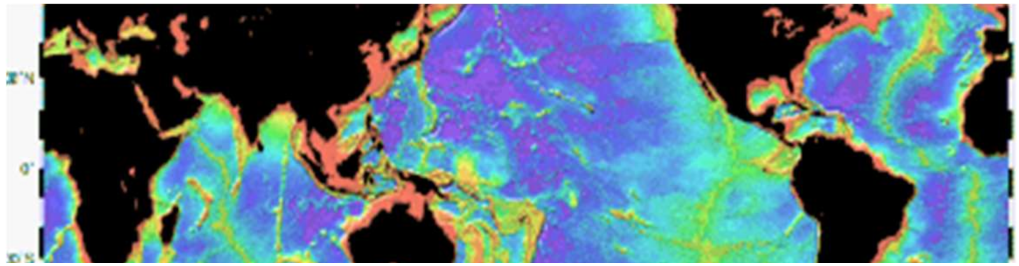
Rossby waves

$$c_p = \frac{\omega}{k} = \bar{U} - \frac{\beta}{K^2 + \frac{f_0^2}{gH}}.$$



Wavelength?
Distance per year?

Do they match?



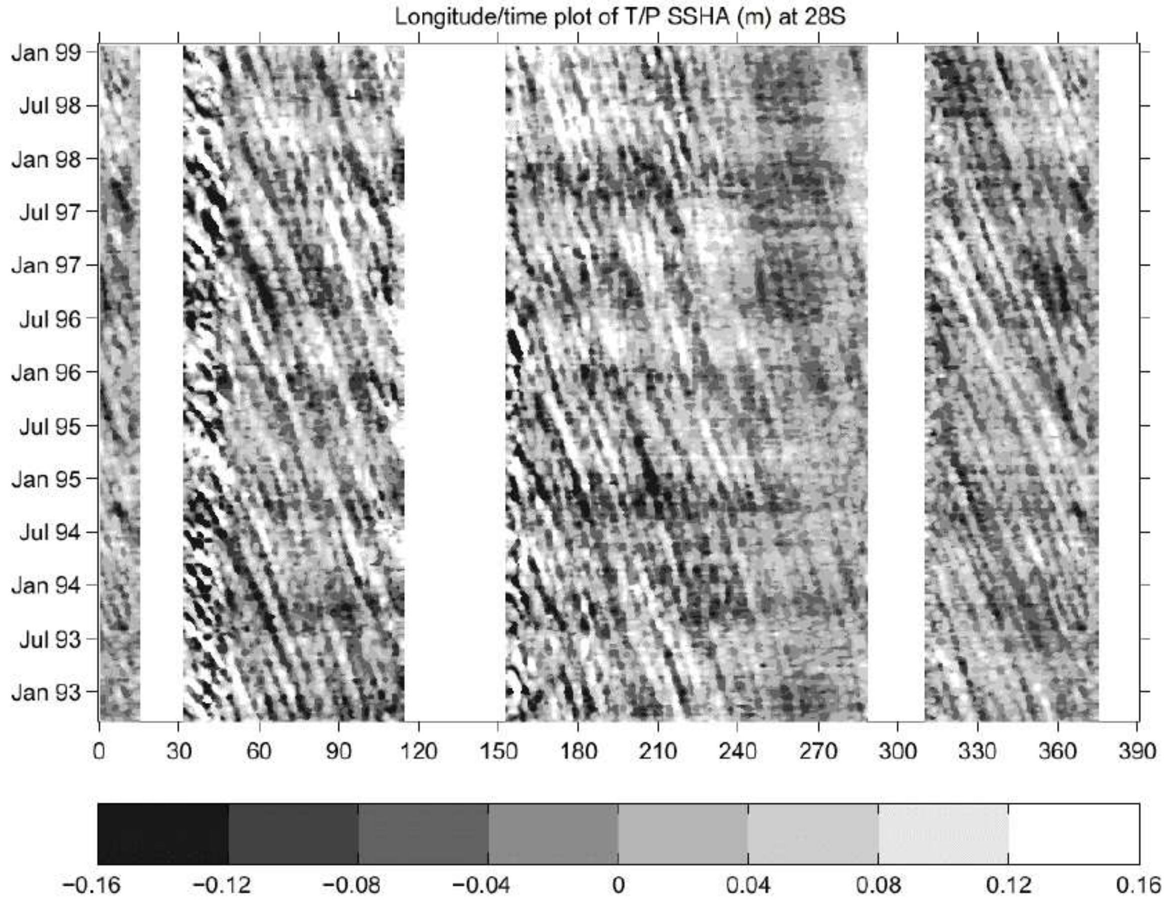
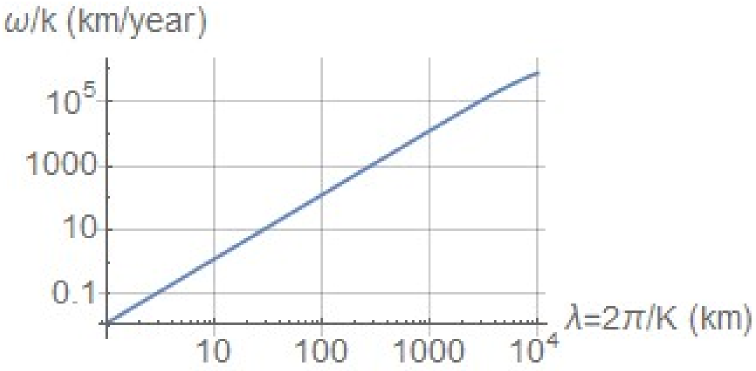


Figure 1 – Global longitude/time plot of Sea Surface Height residuals at 28°S from the T/P altimeter

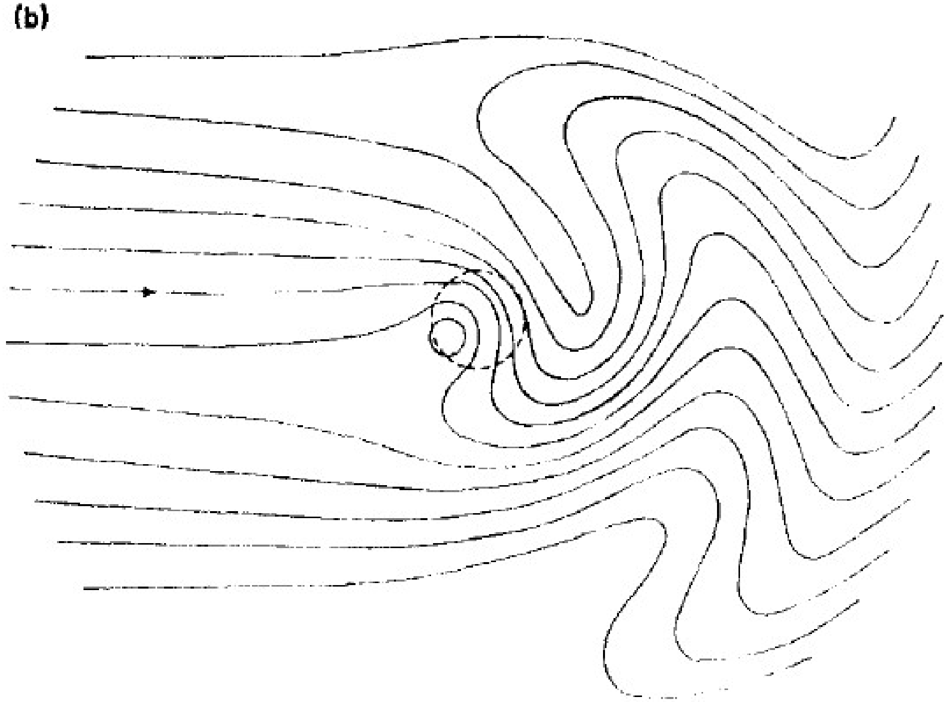
Challenor et al., 2004

Stationary Rossby waves

$$c_p = \frac{\omega}{k} = \bar{U} - \frac{\beta}{K^2 + \frac{f_0^2}{gH}}.$$



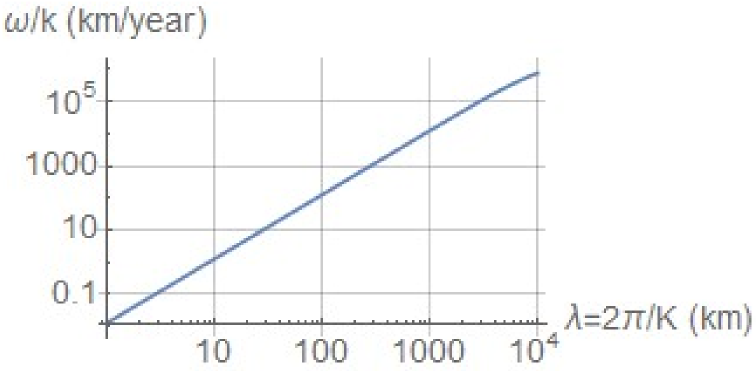
What about if a flow exists such that $c_p = 0$????

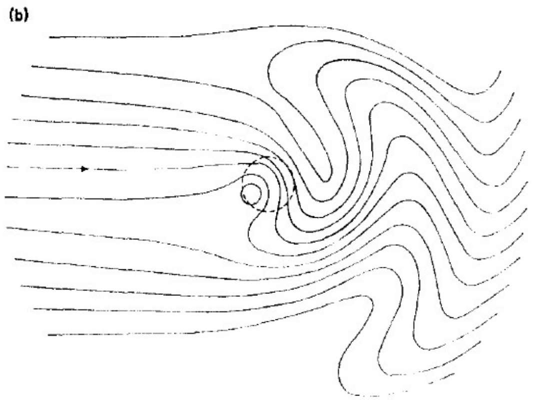


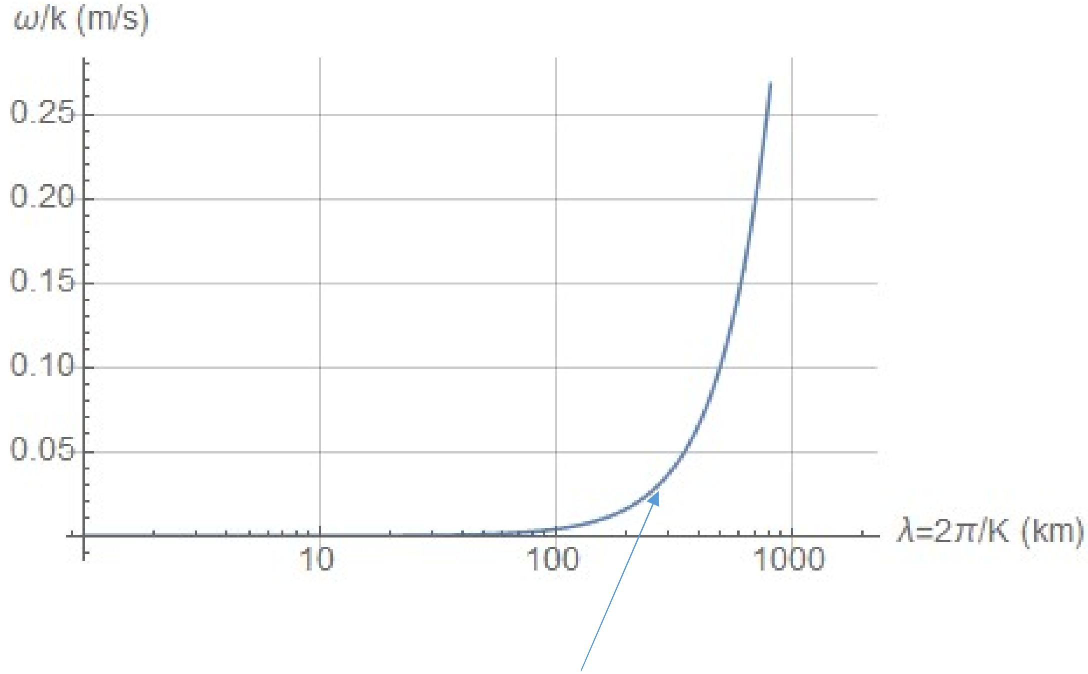
Stationary Rossby waves

$$c_p = \frac{\omega}{k} = \bar{U} - \frac{\beta}{K^2 + \frac{f_0^2}{gH}}.$$

What about if a flow exists such that $c_p = 0$????



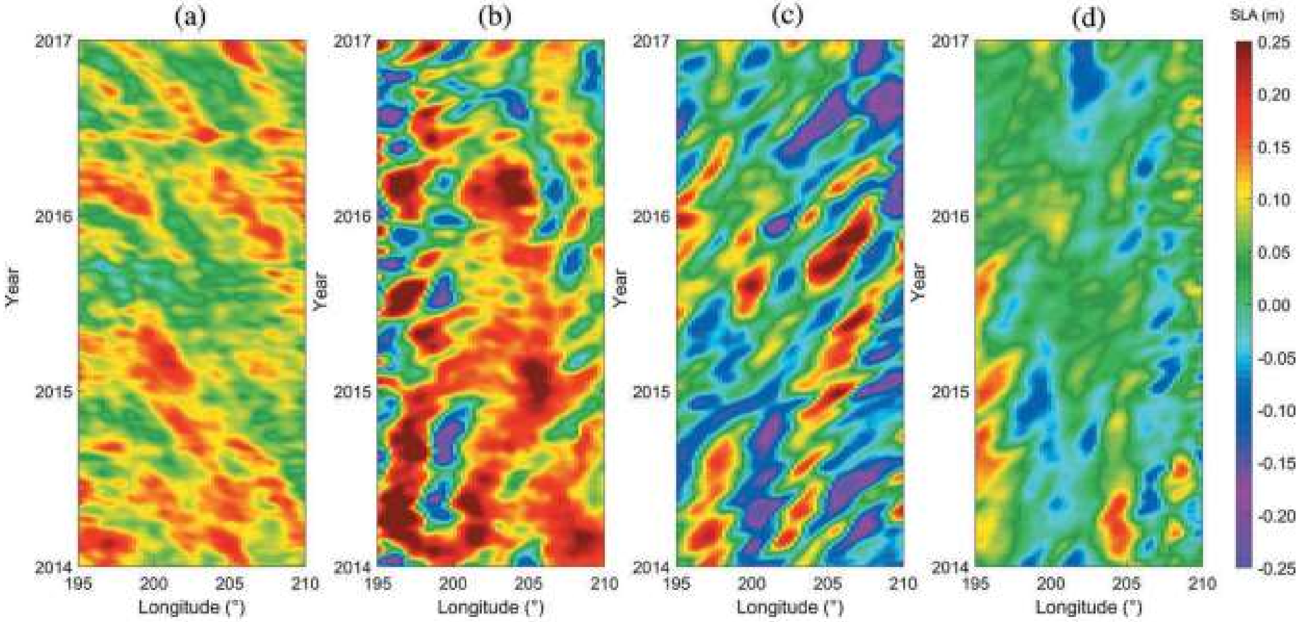




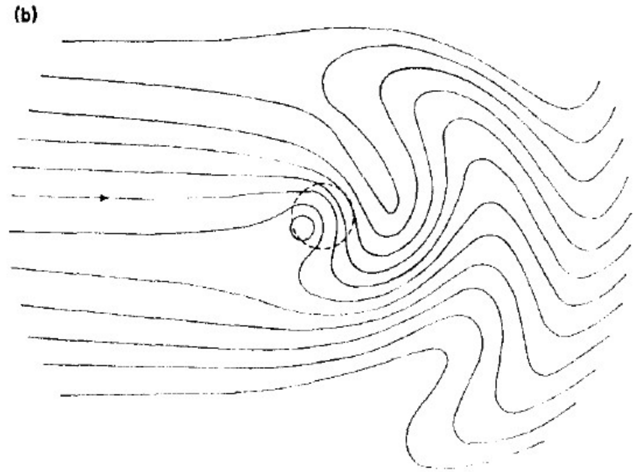
- A westward travelling wave of 200-300km wavelength would be stationary in an eastward flow of a few cm/s
- What is U is faster than this?

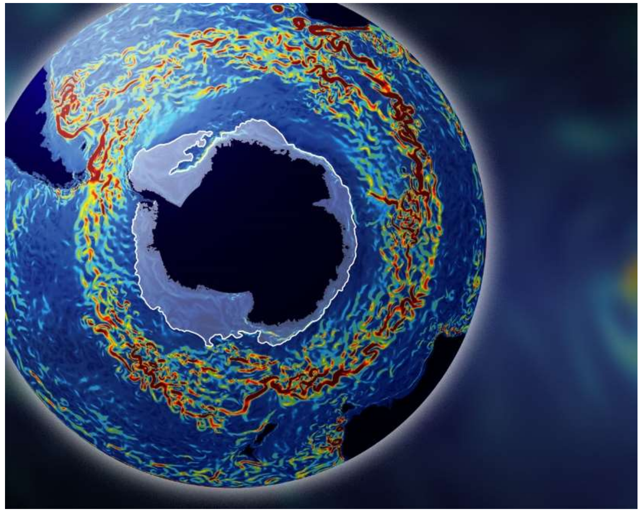
Stationary/trapped Rossby waves

44 49 54 58 deg S

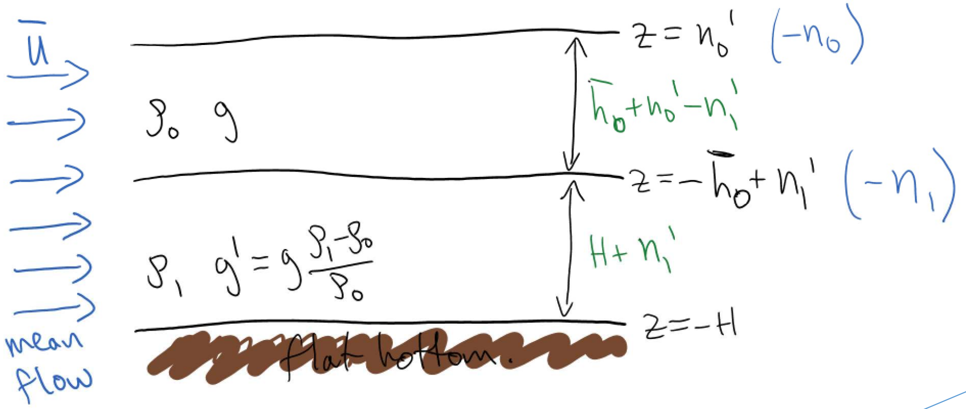


Belonenko et al., 2020





Internal waves



$$\frac{Du_0}{Dt} - fv_0 = g\frac{\partial \eta_0}{\partial x}$$
$$\frac{Dv_0}{Dt} + fu_0 = g\frac{\partial \eta_0}{\partial y}$$
$$\frac{D}{Dt}\left(\frac{\omega_0 + f}{h_0}\right) = 0$$
$$h_0 = \eta_1 - \eta_0$$

$$\frac{Du_1}{Dt} - fv_1 = g\frac{\partial \eta_0}{\partial x} + g'\frac{\partial \eta_1}{\partial x}$$
$$\frac{Dv_1}{Dt} + fu_1 = g\frac{\partial \eta_0}{\partial y} + g'\frac{\partial \eta_1}{\partial y}$$
$$\frac{D}{Dt}\left(\frac{\omega_1 + f}{h_1}\right) = 0$$
$$h_1 = H - \eta_1$$

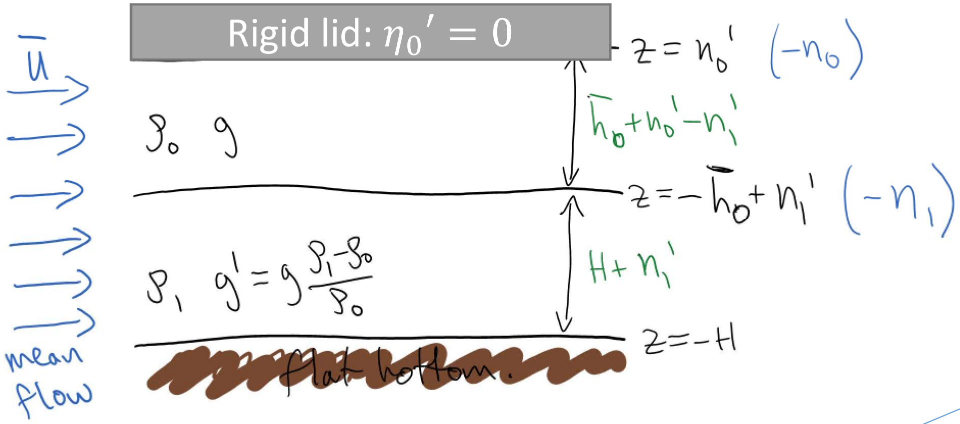
Linearised layer 0

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)u_0 - f_0 v_0 = -g\frac{\partial \eta_0'}{\partial x}$$
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)v_0 + f_0 u_0 = -g\frac{\partial \eta_0'}{\partial y}$$
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_0 - \frac{f_0}{h_0}(\eta_0' - \eta_1')\right) + \beta v_0 = 0$$

Linearised layer 1

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)u_1 - f_0 v_1 = -g\frac{\partial \eta_0'}{\partial x} - g'\frac{\partial \eta_1'}{\partial x}$$
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)v_1 + f_0 u_1 = -g\frac{\partial \eta_0'}{\partial y} - g'\frac{\partial \eta_1'}{\partial y}$$
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_1 - \frac{f_0}{h_1}\eta_1'\right) + \beta v_1 = 0$$

Internal waves



$$\frac{Du_0}{Dt} - fv_0 = g\frac{\partial \eta_0}{\partial x}$$
$$\frac{Dv_0}{Dt} + fu_0 = g\frac{\partial \eta_0}{\partial y}$$
$$\frac{D}{Dt}\left(\frac{\omega_0 + f}{h_0}\right) = 0$$
$$h_0 = \eta_1 - \eta_0$$

$$\frac{Du_1}{Dt} - fv_1 = g\frac{\partial \eta_0}{\partial x} + g'\frac{\partial \eta_1}{\partial x}$$
$$\frac{Dv_1}{Dt} + fu_1 = g\frac{\partial \eta_0}{\partial y} + g'\frac{\partial \eta_1}{\partial y}$$
$$\frac{D}{Dt}\left(\frac{\omega_1 + f}{h_1}\right) = 0$$
$$h_1 = H - \eta_1$$

Linearised layer 0

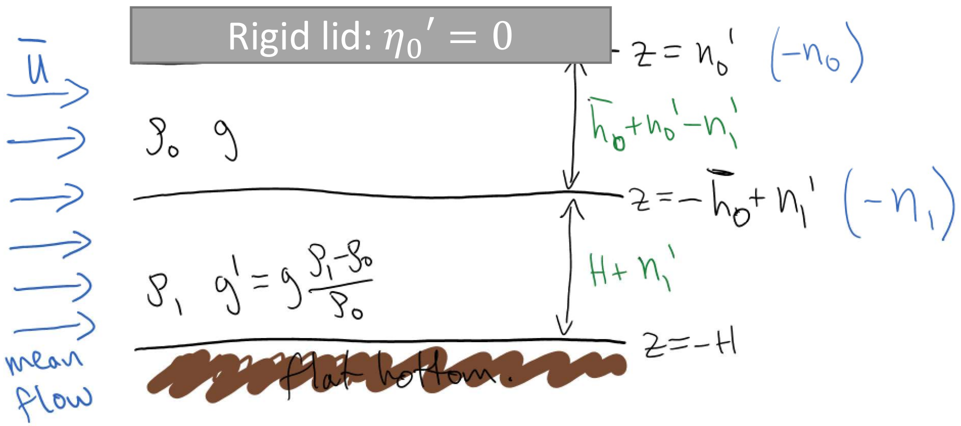
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)u_0 - f_0 v_0 = \cancel{-g\frac{\partial \eta_0'}{\partial x}}$$
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)v_0 + f_0 u_0 = \cancel{-g\frac{\partial \eta_0'}{\partial y}}$$
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_0 - \frac{f_0}{h_0}(\cancel{\eta_0'} - \eta_1')\right) + \beta v_0 = 0$$

Linearised layer 1

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)u_1 - f_0 v_1 = \cancel{-g\frac{\partial \eta_0'}{\partial x}} - g'\frac{\partial \eta_1'}{\partial x}$$
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)v_1 + f_0 u_1 = \cancel{-g\frac{\partial \eta_0'}{\partial y}} - g'\frac{\partial \eta_1'}{\partial y}$$
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_1 - \frac{f_0}{h_1}\eta_1'\right) + \beta v_1 = 0$$

Decouples layer 1 (interior) from layer 0 (surface)

Internal waves



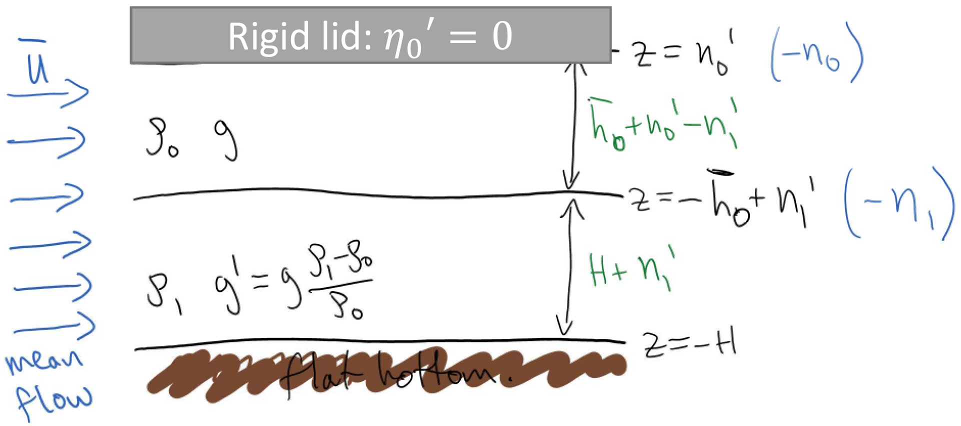
Linearised layer 1 equations with a rigid lid

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)u_1 - f_0 v_1 = -g'\frac{\partial \eta_1'}{\partial x}$$

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)v_1 + f_0 u_1 = -g'\frac{\partial \eta_1'}{\partial y}$$

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_1 - \frac{f_0}{h_1}\eta_1'\right) + \beta v_1 = 0$$

Internal waves



Linearised layer 1 equations with a rigid lid

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$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_1 - \frac{f_0}{h_1}\eta_1'\right) + \beta v_1 = 0$$

Compare with the barotropic equations from before

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)u_0 - f_0 v_0 = -g\frac{\partial \eta_0'}{\partial x}$$

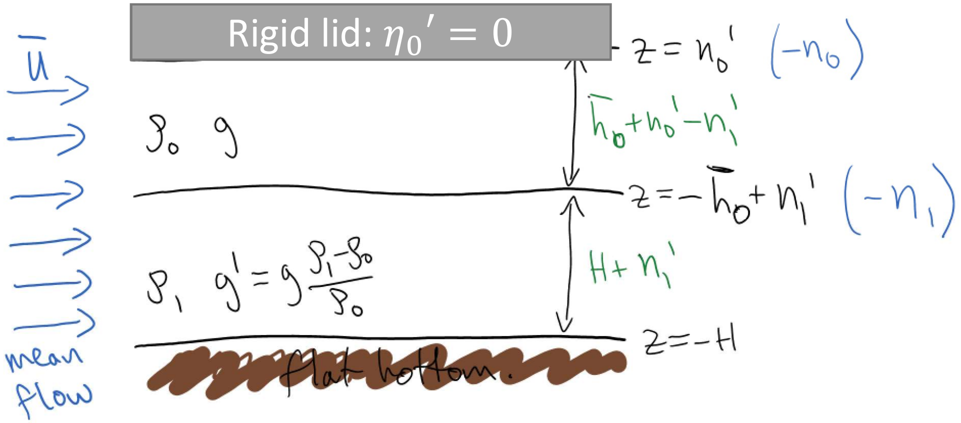
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)v_0 + f_0 u_0 = -g\frac{\partial \eta_0'}{\partial y}$$

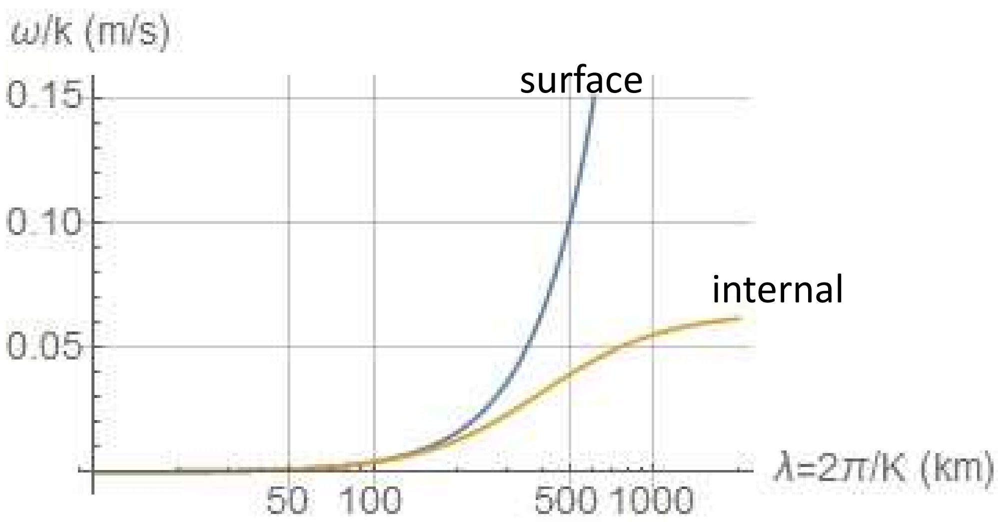
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_0 - \frac{f_0}{H}\eta_0'\right) + \beta v_0 = 0$$

They're the same equations!

The internal waves are the same as the surface waves, but with $g \to g'$ and $H \to h_1$

Internal Rossby waves





Linearised layer 1 equations with a rigid lid

$$\cancel{\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)} u_1 - f_0 v_1 = -g'\frac{\partial \eta_1'}{\partial x}$$

$$\cancel{\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)} v_1 + f_0 u_1 = -g'\frac{\partial \eta_1'}{\partial y}$$

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_1 - \frac{f_0}{h_1}\eta_1'\right) + \beta v_1 = 0$$

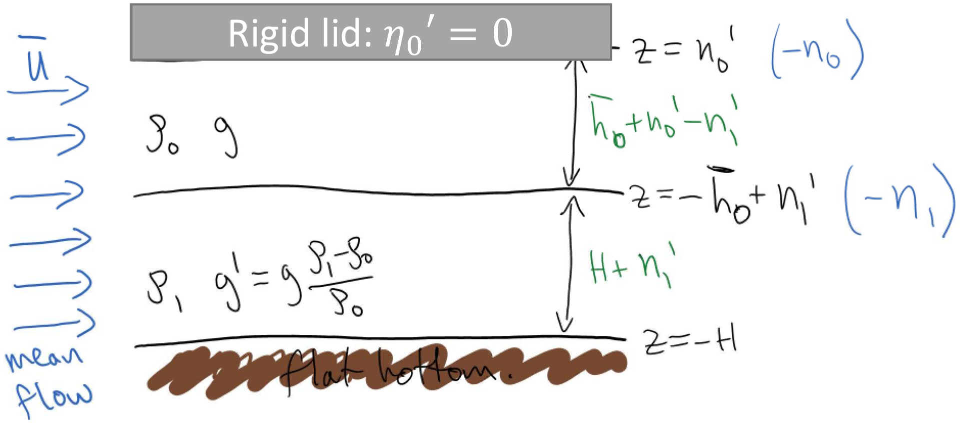
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\frac{g'}{f_0}\nabla^2\eta_1' - \frac{f_0}{h_1}\eta_1'\right) + \frac{g'\beta}{f_0}\frac{\partial \eta_1'}{\partial x} = 0$$

$$c_p = \frac{\omega}{k} = \bar{U} - \frac{\beta}{K^2 + \frac{f_0^2}{g'h_1}}.$$

Internal Rossby waves are SLOWER

For a given flow speed, trapped waves are longer

Internal gravity waves



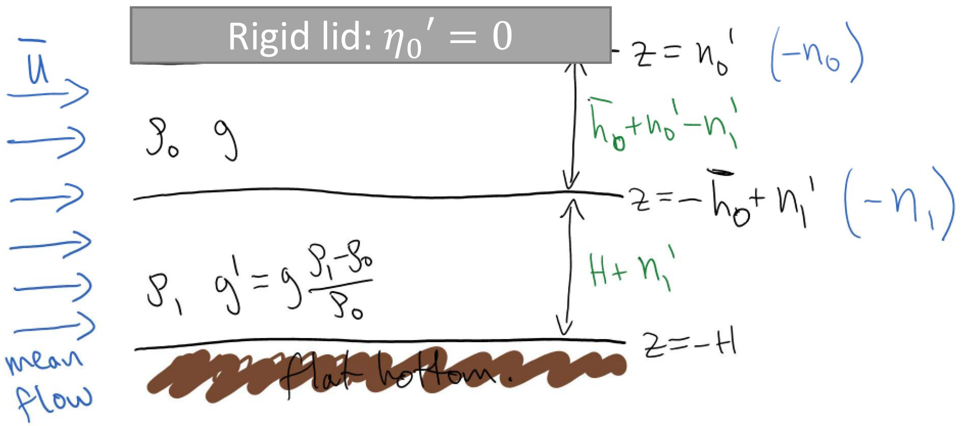
Linearised layer 1 equations with a rigid lid

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)u_1 - f_0 v_1 = -g'\frac{\partial \eta_1'}{\partial x}$$

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)v_1 + f_0 u_1 = -g'\frac{\partial \eta_1'}{\partial y}$$

$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)\left(\omega_1 - \frac{f_0}{h_1}\eta_1'\right) + \cancel{\beta v_1} = 0$$

Internal gravity waves



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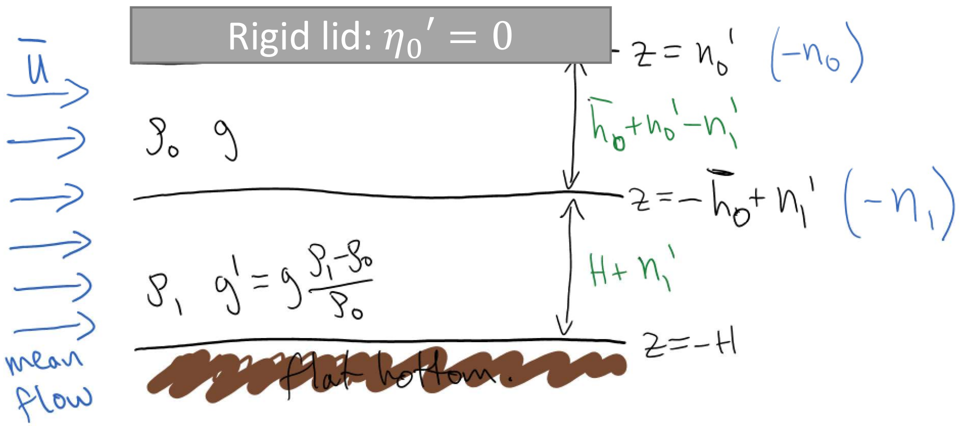
$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)v_1 + f_0 u_1 = -g'\frac{\partial \eta_1'}{\partial y}$$

$$q_1 = \omega_1 - \frac{f_0}{h_1}\eta_1' = \text{const.}$$

Internal waves: balanced wave PV, varying wave momentum
Rossby waves: balanced wave momentum, varying wave PV

Internal gravity waves



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$$\left(\frac{\partial}{\partial t} + \bar{U}\frac{\partial}{\partial x}\right)v_1 + f_0 u_1 = -g'\frac{\partial \eta_1'}{\partial y}$$

$$q_1 = \omega_1 - \frac{f_0}{h_1}\eta_1' = \text{const.}$$

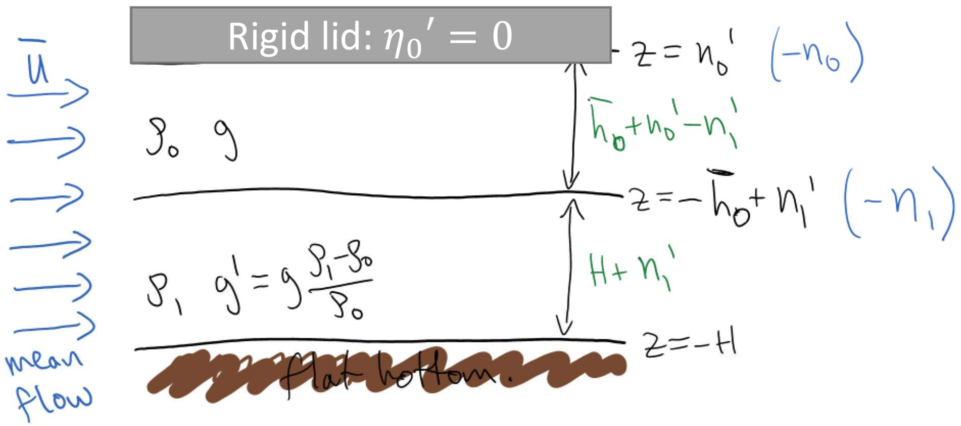
Let $\phi = \hat{\phi}\, e^{i(kx+ly-\omega t)}$

$$(-i\omega + ik\bar{u})\hat{u}_1 - f_0 \hat{v}_1 = -ikg'\hat{\eta}_1'$$

$$(-i\omega + ik\bar{u})\hat{v}_1 + f_0 \hat{u}_1 = -ilg'\hat{\eta}_1'$$

$$ik\hat{v}_1 - il\hat{u}_1 - \frac{f_0}{h_1}\hat{\eta}_1' = 0$$

Internal gravity waves



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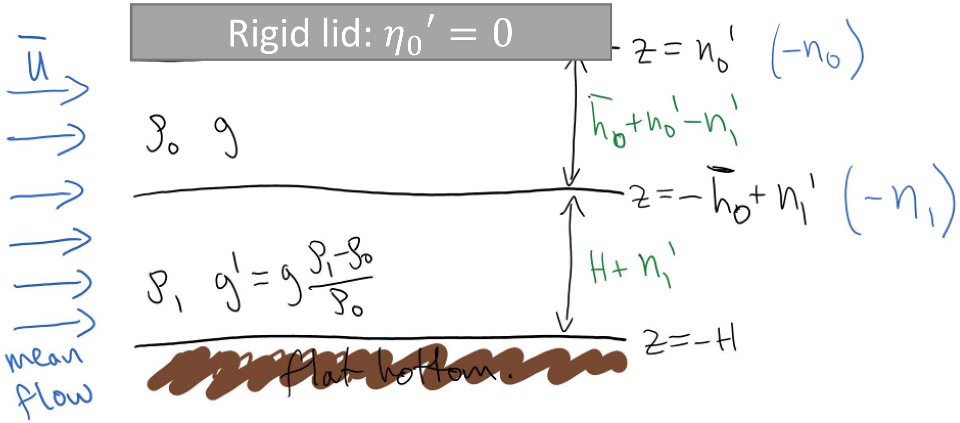
$$ik\hat{v}_1 - il\hat{u}_1 - \frac{f_0}{h_1}\hat{\eta}_1' = 0$$

Etc, etc, etc, ...

$$\omega = k\bar{U} \pm \sqrt{f_0^2 + K^2 g' h_1}$$

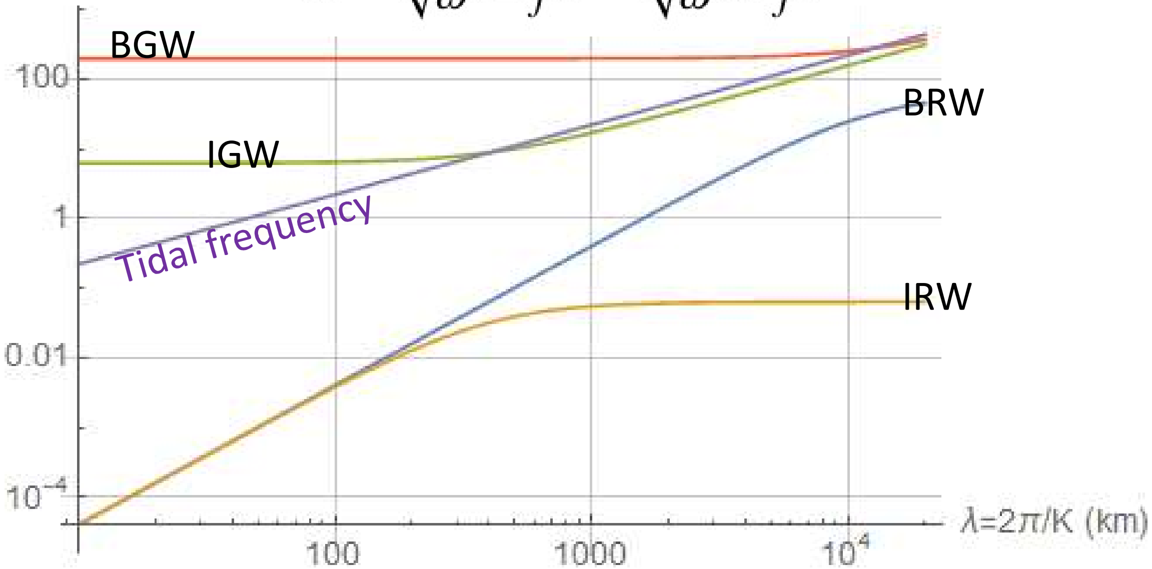
Internal gravity wave dispersion relation

Internal gravity waves



IGWs are MUCH smaller than BGWs

$$\lambda = \frac{2\pi}{K} = \frac{2\pi g' h_1}{\sqrt{\omega^2 - f^2}} \ll \frac{2\pi g H}{\sqrt{\omega^2 - f^2}}$$



$$\left(\frac{\partial}{\partial t} + \bar U \frac{\partial}{\partial x}\right) u_1 - f_0 v_1 = -g' \frac{\partial \eta_1'}{\partial x}$$

$$\left(\frac{\partial}{\partial t} + \bar U \frac{\partial}{\partial x}\right) v_1 + f_0 u_1 = -g' \frac{\partial \eta_1'}{\partial y}$$

$$q_1 = \omega_1 - \frac{f_0}{h_1}\eta_1' = \text{const.}$$

Let $\phi = \hat\phi e^{i(kx+ly-\omega t)}$

$$(-i\omega + ik\bar U)\hat u_1 - f_0 \hat v_1 = -ikg'\hat\eta_1'$$

$$(-i\omega + ik\bar U)\hat v_1 + f_0 \hat u_1 = -ilg'\hat\eta_1'$$

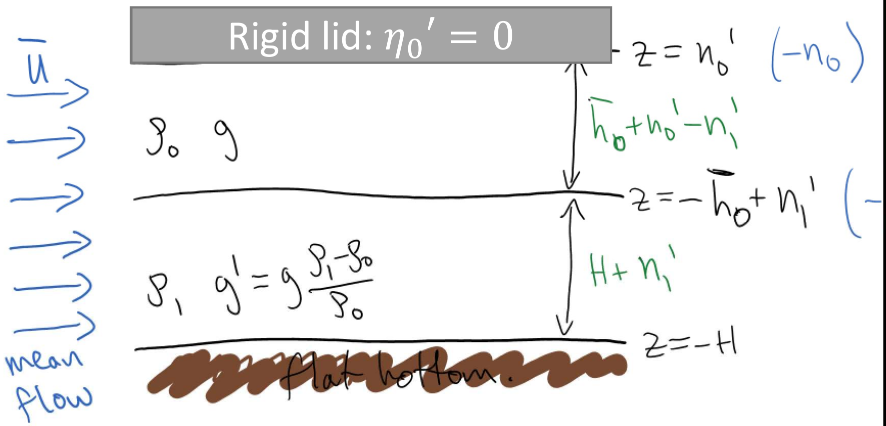
$$ik\hat v_1 - il\hat u_1 - \frac{f_0}{h_1}\hat\eta_1' = 0$$

Etc, etc, etc, ………….

$$\omega = k\bar U \pm \sqrt{f_0^2 + K^2 g' h_1}$$

Internal gravity wave dispersion relation

Internal gravity waves



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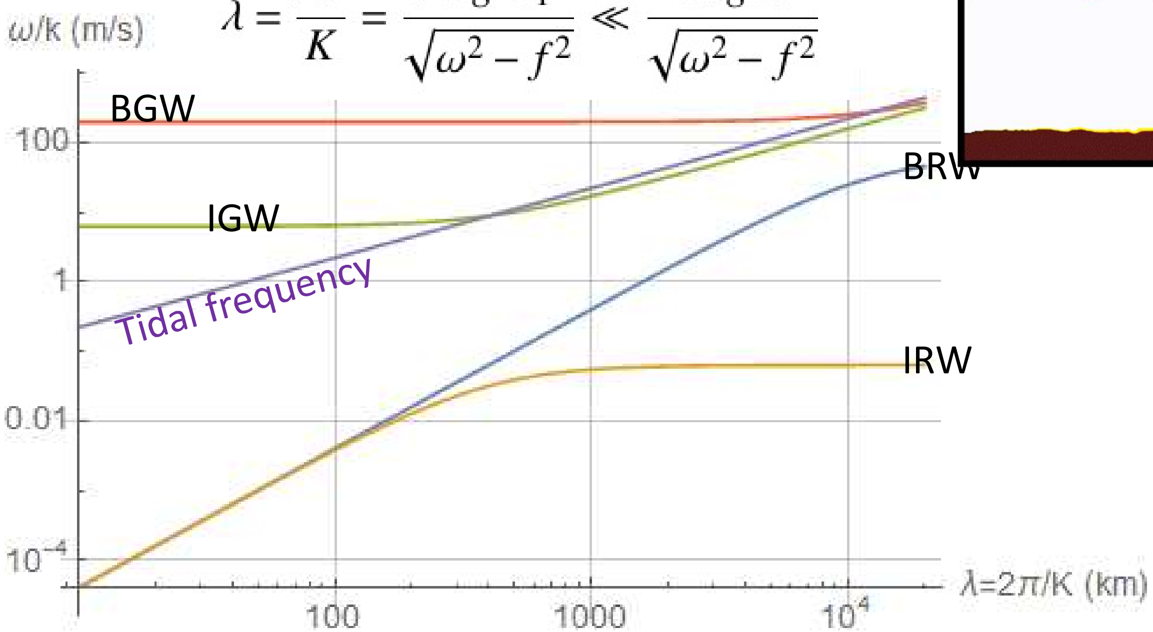
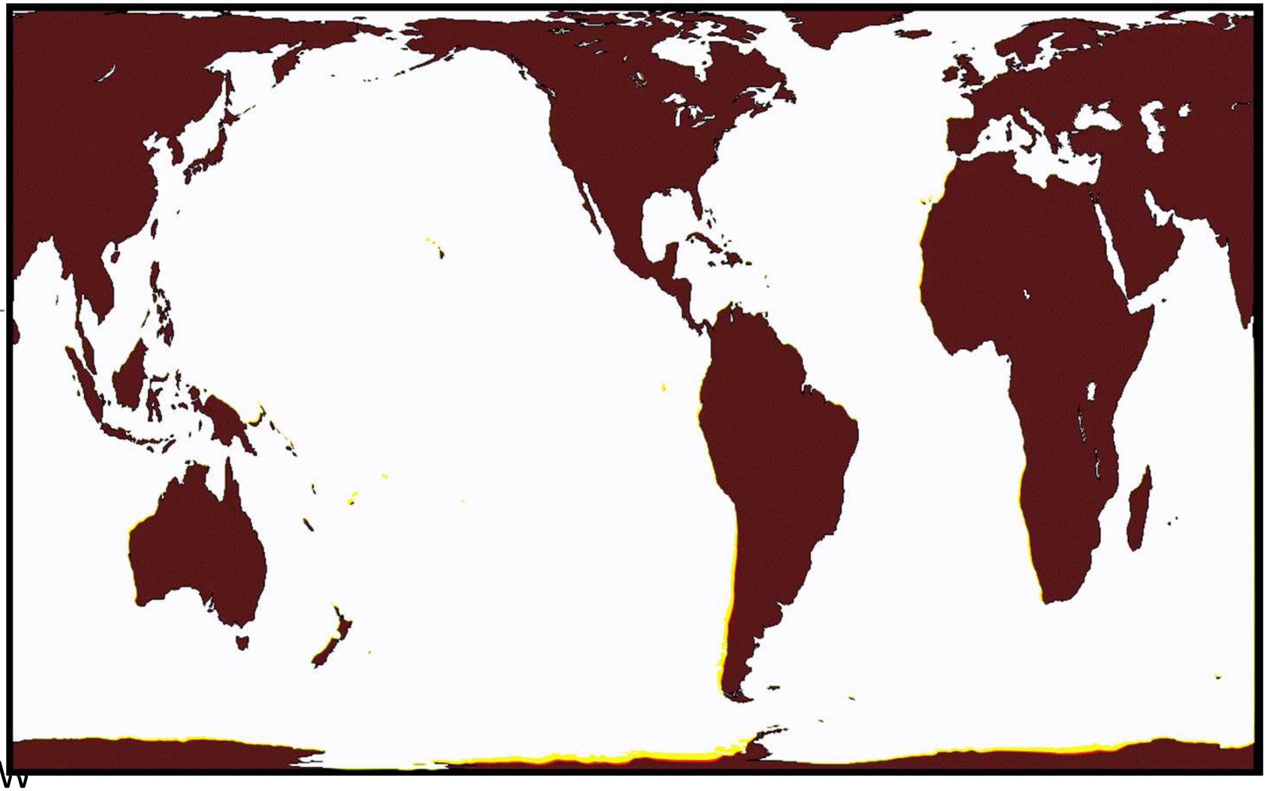
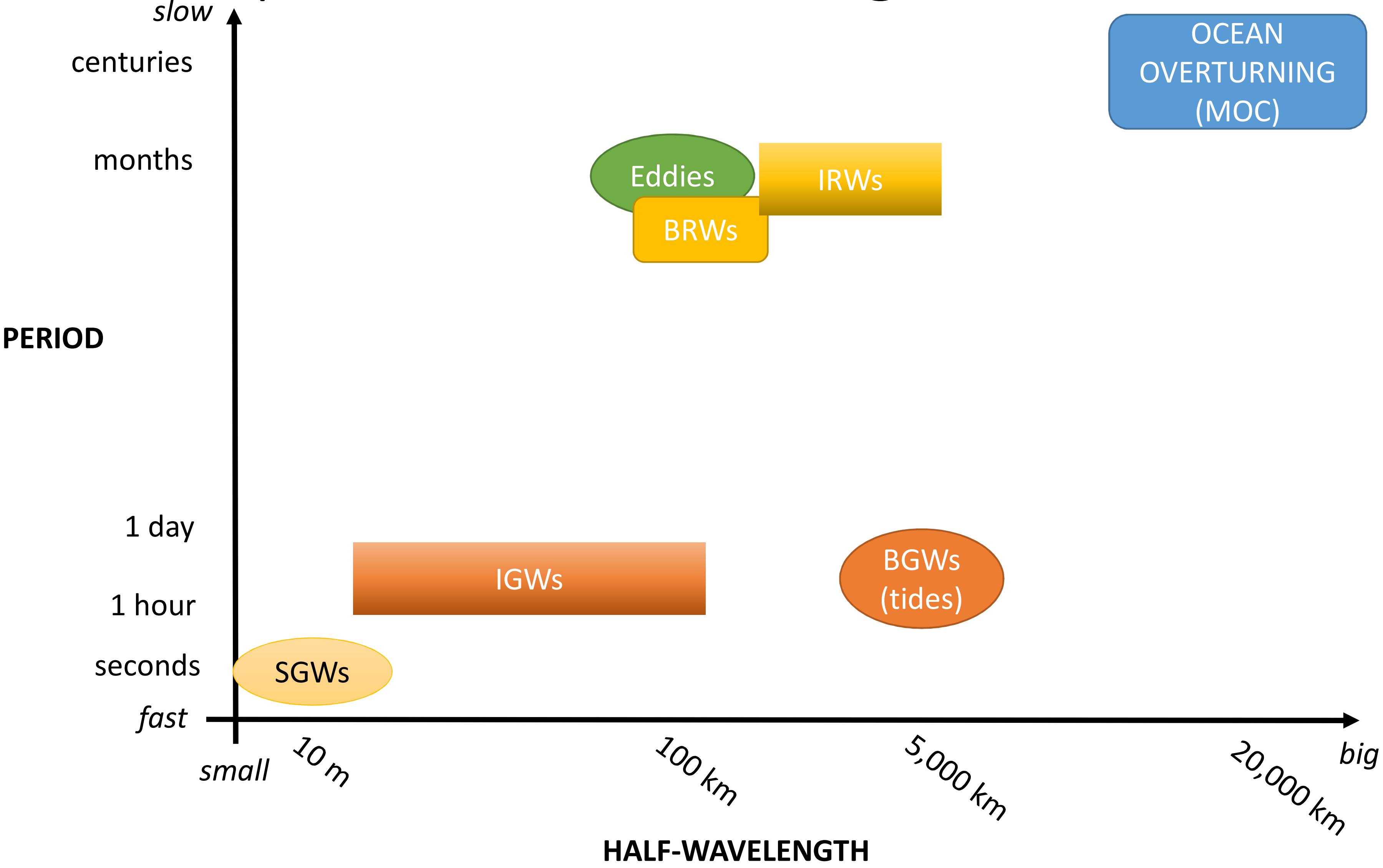




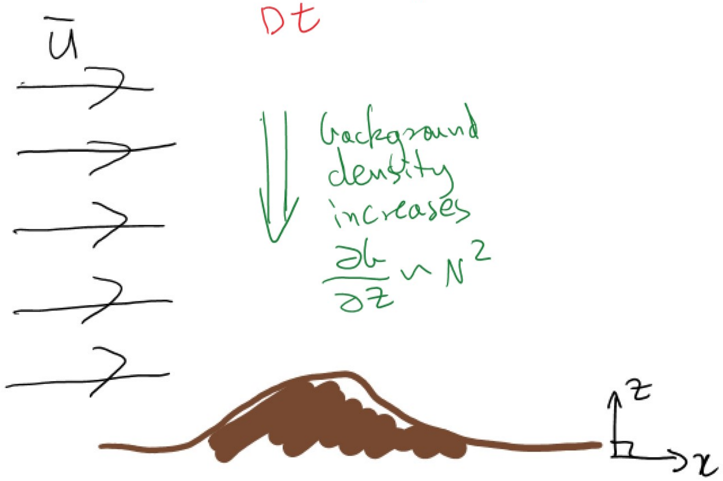
Simmons et al., 2004

Ocean space-time scale diagram

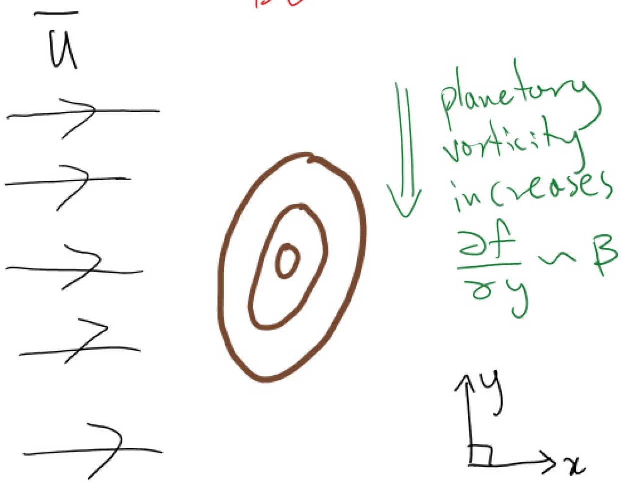


The mechanism behind the waves

Gravity waves $\frac{Db'}{Dt} = -N^2 w'$



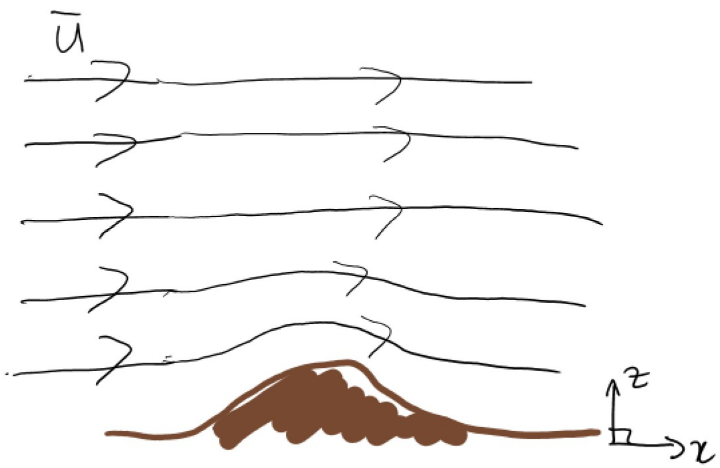
Rossby waves $\frac{D\omega'}{Dt} = -\beta v'$



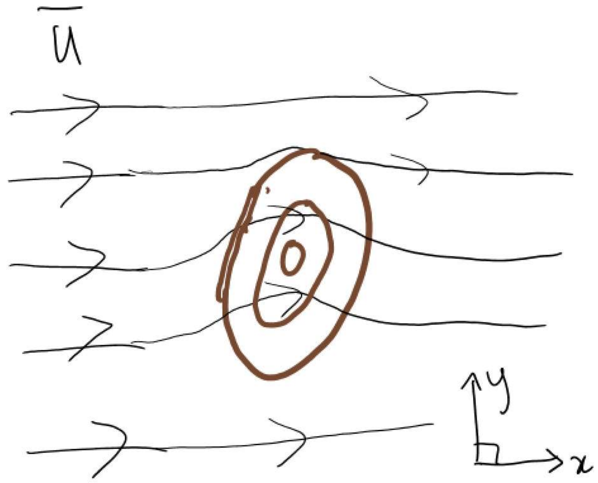
- The bathymetry induces a z (or y) velocity to the flow, which reduces its buoyancy (or vorticity)
- If the perturbation is slow/weak, the flow remains in/near balance and returns to its original course….

The mechanism behind the waves

Gravity waves



Rossby waves



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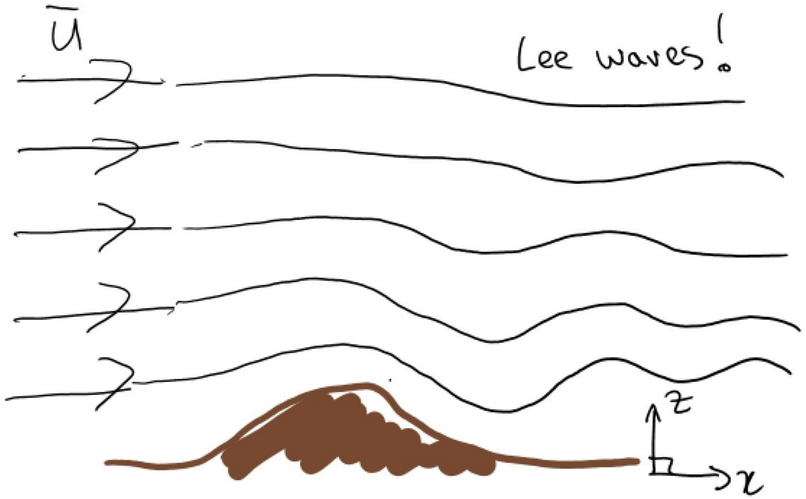
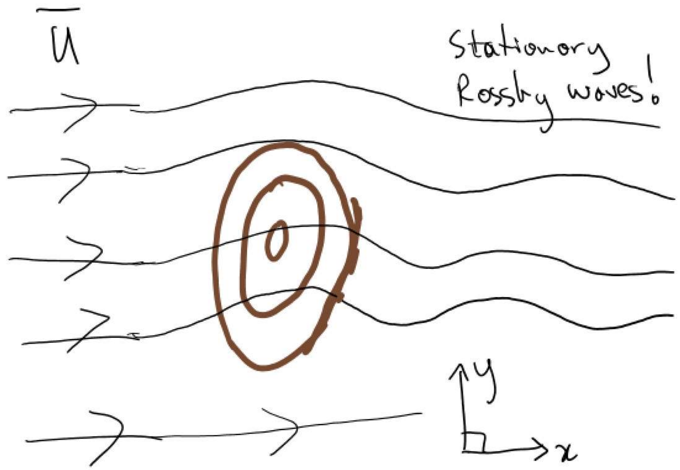
The mechanism behind the waves

Gravity waves



Rossby waves



- The bathymetry induces a z (or y) velocity to the flow, which reduces its buoyancy (or vorticity)
- If the perturbation is slow/weak, the flow remains in/near balance and returns to its original course….
- But if the perturbation is **fast/strong** ($\bar{U} \sim \frac{\omega}{k}$), it kicks of an oscillation in the lee of the obstacle…

References

- Challenor, Peter G., et al. "Characteristics of mid-latitude Rossby wave propagation from multiple satellite datasets." *International Journal of Remote Sensing* 25.7-8 (2004): 1297-1302.
- Belonenko, Tatyana, Anastasia Frolova, and Vladimir Gnevyshev. "Detection of waveguide for Rossby waves using satellite altimetry in the Antarctic Circumpolar Current." *International Journal of Remote Sensing* 41.16 (2020): 6232-6247.
- Simmons, Harper L., Robert W. Hallberg, and Brian K. Arbic. "Internal wave generation in a global baroclinic tide model." *Deep Sea Research Part II: Topical Studies in Oceanography* 51.25-26 (2004): 3043-3068.